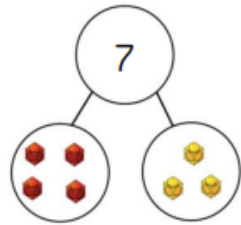


At Kobi Nazrul, we teach to ensure understanding of number. We actively look for patterns and use these representatives to support this.

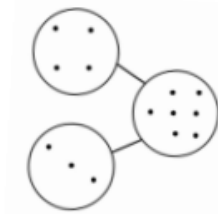
Here is a list of models we use to support the understanding of number. They can also be adapted to include measures. Children build fluency of these models and can then choose which one make most sense to them.

Part-Whole Model

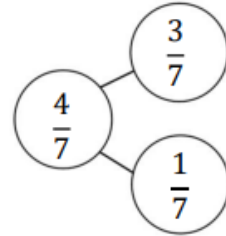
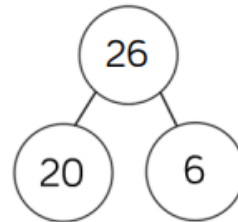
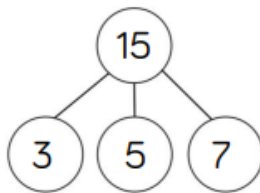
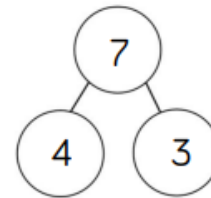
Addition and subtraction



$$7 = 4 + 3$$
$$7 = 3 + 4$$



$$7 - 3 = 4$$
$$7 - 4 = 3$$



Benefits

This part-whole model supports children in their understanding of aggregation and partitioning. Due to its shape, it can be referred to as a cherry part-whole model.

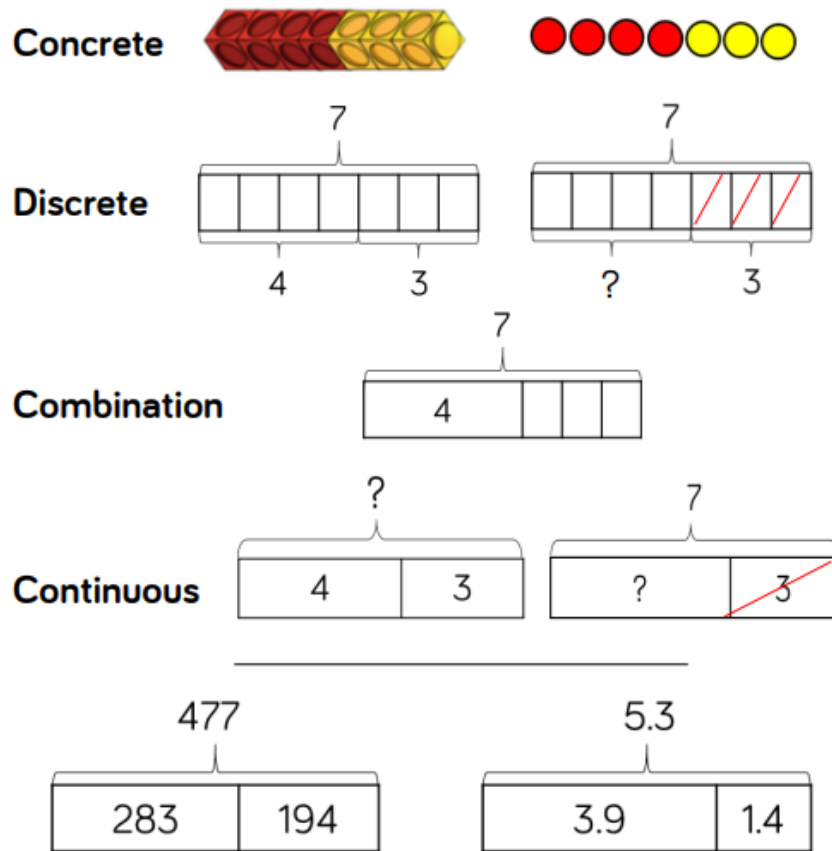
When the parts are complete and the whole is empty, children use aggregation to add the parts together to find the total.

When the whole is complete and at least one of the parts is empty, children use partitioning (a form of subtraction) to find the missing part.

Part-whole models can be used to partition a number into two or more parts, or to help children to partition a number into tens and ones or other place value columns.

In KS2, children can apply their understanding of the part-whole model to add and subtract fractions, decimals and percentages.

Bar Model (single)



Benefits

The single bar model is another type of a part-whole model that can support children in representing calculations to help them unpick the structure.

Cubes and counters can be used in a line as a concrete representation of the bar model.

Discrete bar models are a good starting point with smaller numbers. Each box represents one whole.

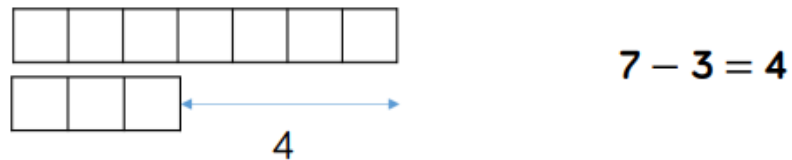
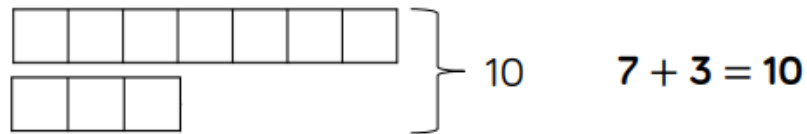
The combination bar model can support children to calculate by counting on from the larger number. It is a good stepping stone towards the continuous bar model.

Continuous bar models are useful for a range of values. Each rectangle represents a number. The question mark indicates the value to be found.

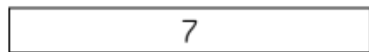
In KS2, children can use bar models to represent larger numbers, decimals and fractions.

Bar Model (multiple)

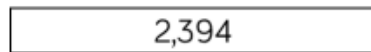
Discrete



Continuous



$$7 - 3 = 4$$



$$2,394 - 1,014 = 1,380$$

Benefits

The multiple bar model is a good way to compare quantities whilst still unpicking the structure.

Two or more bars can be drawn, with a bracket labelling the whole positioned on the right hand side of the bars. Smaller numbers can be represented with a discrete bar model whilst continuous bar models are more effective for larger numbers.

Multiple bar models can also be used to represent the difference in subtraction. An arrow can be used to model the difference.

When working with smaller numbers, children can use cubes and a discrete model to find the difference. This supports children to see how counting on can help when finding the difference.

Number Shapes



$7 = 4 + 3$



$7 = 3 + 4$



$7 - 3 = 4$



$6 + 4$



$7 + 3$



$8 + 2$



$9 + 1$

Benefits

Number shapes can be useful to support children to subitise numbers as well as explore aggregation, partitioning and number bonds.

When adding numbers, children can see how the parts come together making a whole. As children use number shapes more often, they can start to subitise the total due to their familiarity with the shape of each number.

When subtracting numbers, children can start with the whole and then place one of the parts on top of the whole to see what part is missing. Again, children will start to be able to subitise the part that is missing due to their familiarity with the shapes.

Children can also work systematically to find number bonds. As they increase one number by 1, they can see that the other number decreases by 1 to find all the possible number bonds for a number.

Cubes



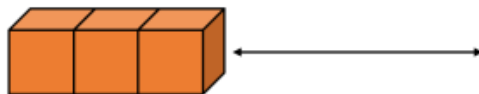
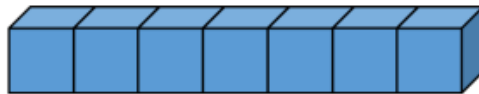
$$7 = 4 + 3$$



$$7 = 3 + 4$$



$$7 - 3 = 4$$



$$7 - 3 = 4$$

Benefits

Cubes can be useful to support children with the addition and subtraction of one-digit numbers.

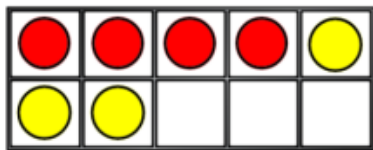
When adding numbers, children can see how the parts come together to make a whole. Children could use two different colours of cubes to represent the numbers before putting them together to create the whole.

When subtracting numbers, children can start with the whole and then remove the number of cubes that they are subtracting in order to find the answer. This model of subtraction is reduction, or take away.

Cubes can also be useful to look at subtraction as difference. Here, both numbers are made and then lined up to find the difference between the numbers.

Cubes are useful when working with smaller numbers but are less efficient with larger numbers as they are difficult to subitise and children may miscount them.

Ten Frames (within 10)



$$4 + 3 = 7$$

4 is a part.

$$3 + 4 = 7$$

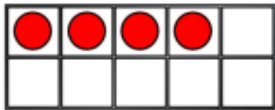
3 is a part.

$$7 - 3 = 4$$

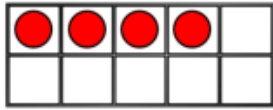
7 is the whole.

$$7 - 4 = 3$$

First

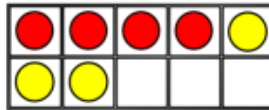


Then

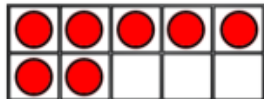


$$4 + 3 = 7$$

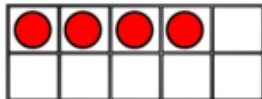
Now



First

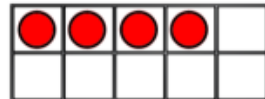


Then



$$7 - 3 = 4$$

Now



Benefits

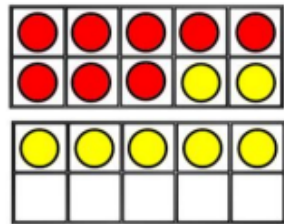
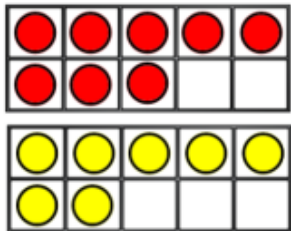
When adding and subtracting within 10, the ten frame can support children to understand the different structures of addition and subtraction.

Using the language of parts and wholes represented by objects on the ten frame introduces children to aggregation and partitioning.

Aggregation is a form of addition where parts are combined together to make a whole. Partitioning is a form of subtraction where the whole is split into parts. Using these structures, the ten frame can enable children to find all the number bonds for a number.

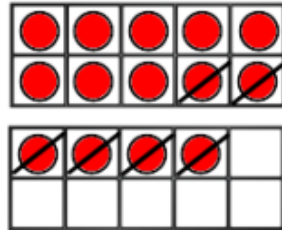
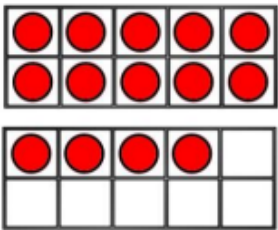
Children can also use ten frames to look at augmentation (increasing a number) and take-away (decreasing a number). This can be introduced through a first, then, now structure which shows the change in the number in the 'then' stage. This can be put into a story structure to help children understand the change e.g. First, there were 7 cars. Then, 3 cars left. Now, there are 4 cars.

Ten Frames (within 20)



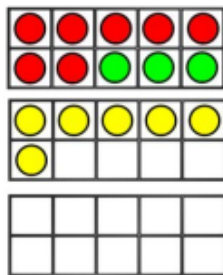
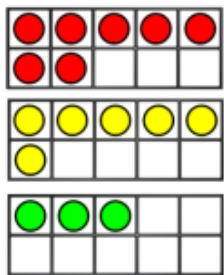
$$8 + 7 = 15$$

Diagram showing 8 and 7 being partitioned to make 10 (2 from 8 and 8 from 7) and 5 remaining from 7.



$$14 - 6 = 8$$

Diagram showing 14 and 6 being partitioned to make 10 (4 from 14 and 6 from 6) and 8 remaining from 14.



$$7 + 6 + 3 = 16$$

Diagram showing 7, 6, and 3 being partitioned to make 10 (2 from 7 and 8 from 6) and 3 remaining from 3.

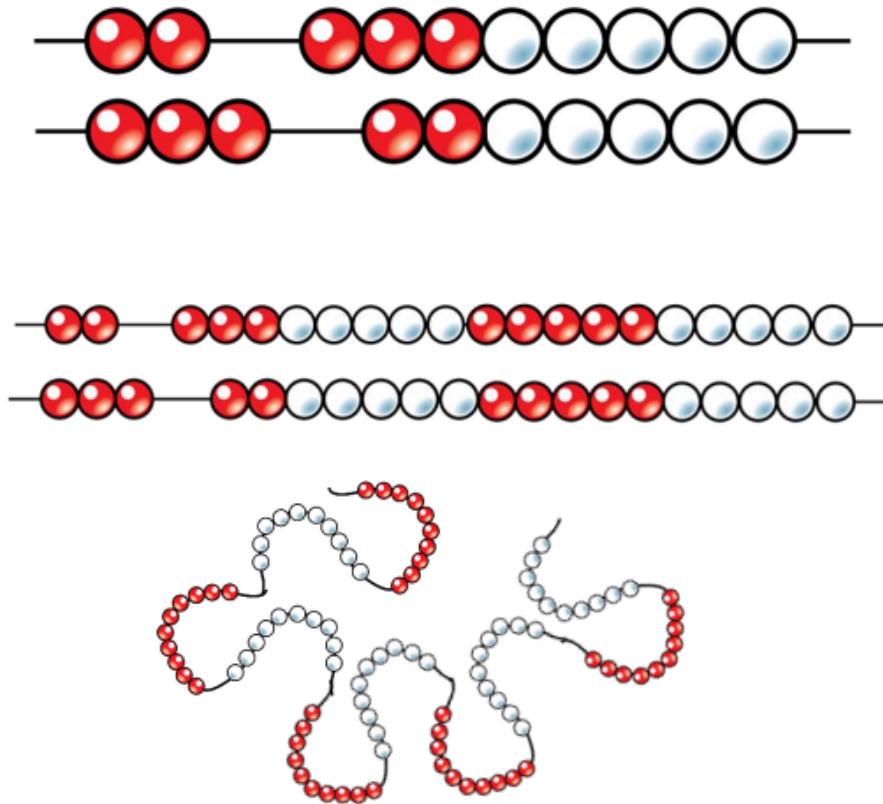
Benefits

When adding two single digits, children can make each number on separate ten frames before moving part of one number to make 10 on one of the ten frames. This supports children to see how they have partitioned one of the numbers to make 10, and makes links to effective mental methods of addition.

When subtracting a one-digit number from a two-digit number, firstly make the larger number on 2 ten frames. Remove the smaller number, thinking carefully about how you have partitioned the number to make 10, this supports mental methods of subtraction.

When adding three single-digit numbers, children can make each number on 3 separate 10 frames before considering which order to add the numbers in. They may be able to find a number bond to 10 which makes the calculation easier. Once again, the ten frames support the link to effective mental methods of addition as well as the importance of commutativity.

Bead Strings



Benefits

Different sizes of bead strings can support children at different stages of addition and subtraction.

Bead strings to 10 are very effective at helping children to investigate number bonds up to 10.

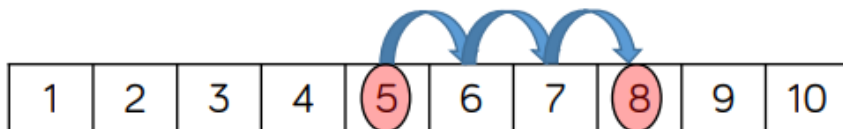
They can help children to systematically find all the number bonds to 10 by moving one bead at a time to see the different numbers they have partitioned the 10 beads into e.g. $2 + 8 = 10$, move one bead, $3 + 7 = 10$.

Bead strings to 20 work in a similar way but they also group the beads in fives. Children can apply their knowledge of number bonds to 10 and see the links to number bonds to 20.

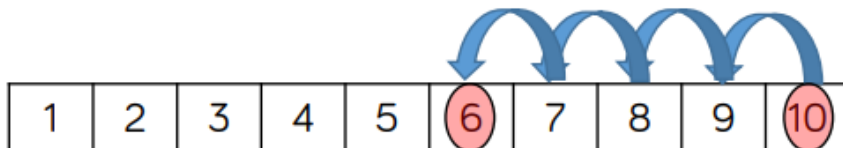
Bead strings to 100 are grouped in tens and can support children in number bonds to 100 as well as helping when adding by making ten. Bead strings can show a link to adding to the next 10 on number lines which supports a mental method of addition.

Number Tracks

$$5 + 3 = 8$$



$$10 - 4 = 6$$



$$8 + 7 = 15$$



This is a subtle pre-step to the number line which allows children to put counters on the number inside a box to count on or back from. It supports seeing each number in its own box. This allows children to move counters along the track – board games with numbers inside boxes is a good way of consolidating this.

Benefits

Number tracks are useful to support children in their understanding of augmentation and reduction.

When adding, children count on to find the total of the numbers. On a number track, children can place a counter on the starting number and then count on to find the total.

When subtracting, children count back to find their answer. They start at the minuend and then take away the subtrahend to find the difference between the numbers.

Number tracks can work well alongside ten frames and bead strings which can also model counting on or counting back.

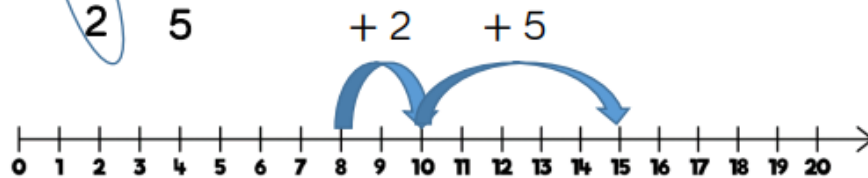
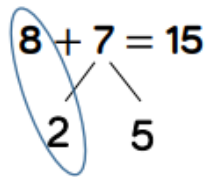
Playing board games can help children to become familiar with the idea of counting on using a number track before they move on to number lines.

Number Lines (labelled)

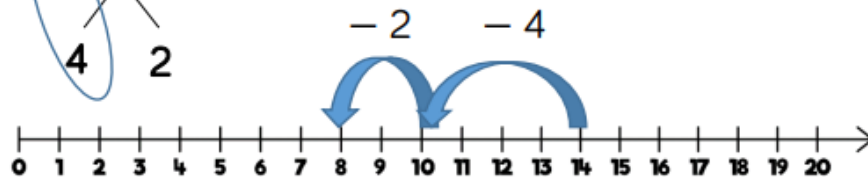
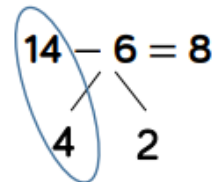
$$5 + 3 = 8$$



$$8 + 7 = 15$$



$$14 - 6 = 8$$



Benefits

Labelled number lines support children in their understanding of addition and subtraction as augmentation and reduction.

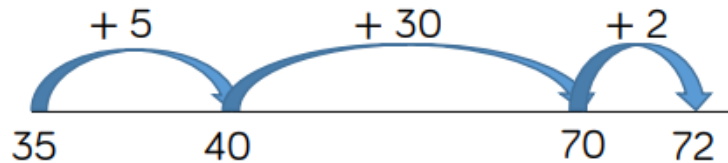
Children can start by counting on or back in ones, up or down the number line. This skill links directly to the use of the number track.

Progressing further, children can add numbers by jumping to the nearest 10 and then jumping to the total. This links to the making 10 method which can also be supported by ten frames. The smaller number is partitioned to support children to make a number bond to 10 and to then add on the remaining part.

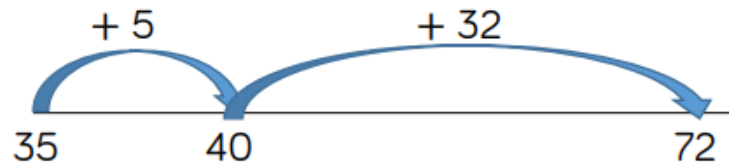
Children can subtract numbers by firstly jumping to the nearest 10. Again, this can be supported by ten frames so children can see how they partition the smaller number into the two separate jumps.

Number Lines (blank)

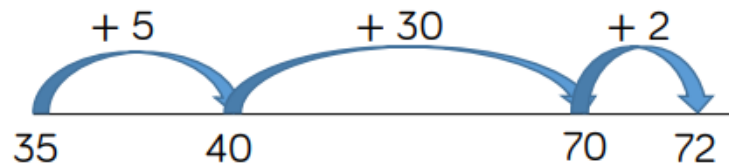
$$35 + 37 = 72$$



$$35 + 37 = 72$$



$$72 - 35 = 37$$



Benefits

Blank number lines provide children with a structure to add and subtract numbers in smaller parts.

Developing from labelled number lines, children can add by jumping to the nearest 10 and then adding the rest of the number either as a whole or by adding the tens and ones separately.

Children may also count back on a number line to subtract, again by jumping to the nearest 10 and then subtracting the rest of the number.

Blank number lines can also be used effectively to help children subtract by finding the difference between numbers. This can be done by starting with the smaller number and then counting on to the larger number. They then add up the parts they have counted on to find the difference between the numbers.

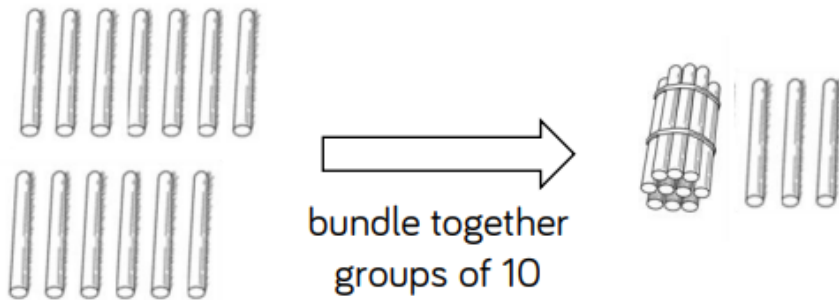
These can also be done in reverse with children developing their fluency of counting back. For some children using addition to solve a subtraction is conceptually muddling until secure understanding of the relationship between addition and subtraction.

$72 - 35$ can be done by partitioning 35 into 2 (to get to the nearest 10) 3, 30.

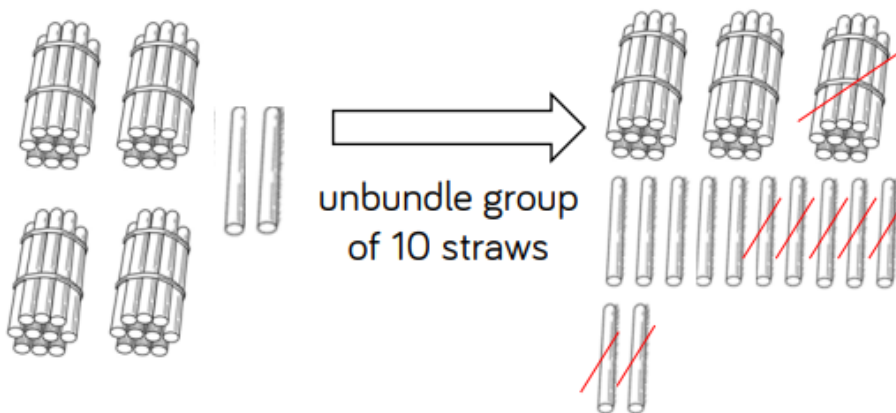
This can also be done in jumps of 10s and 1s to scaffold for those developing place value understanding.

Straws

$$7 + 6 = 13$$



$$42 - 17 = 25$$



Benefits

Straws are an effective way to support children in their understanding of exchange when adding and subtracting 2-digit numbers.

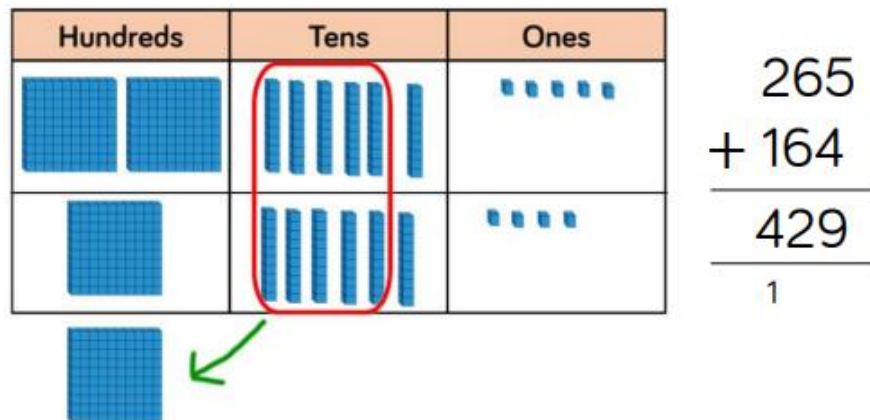
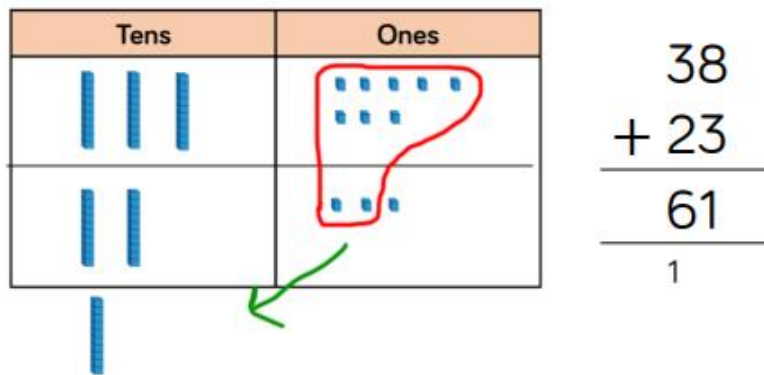
Children can be introduced to the idea of bundling groups of ten when adding smaller numbers and when representing 2-digit numbers. Use elastic bands or other ties to make bundles of ten straws.

When adding numbers, children bundle a group of 10 straws to represent the exchange from 10 ones to 1 ten. They then add the individual straws (ones) and bundles of straws (tens) to find the total.

When subtracting numbers, children unbundle a group of 10 straws to represent the exchange from 1 ten to 10 ones.

Straws provide a good stepping stone to adding and subtracting with Base 10/Dienes.

Base 10/Dienes (addition)



Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

Children should first add without an exchange before moving on to addition with exchange. The representation becomes less efficient with larger numbers due to the size of Base 10. In this case, place value counters may be the better model to use.

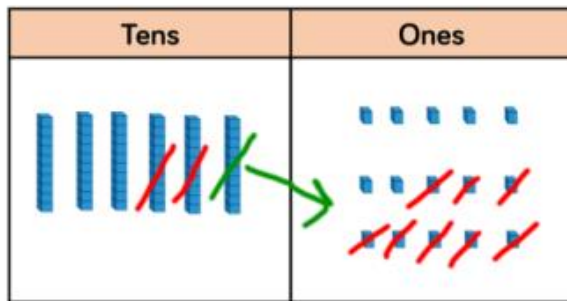
When adding, always start with the smallest place value column. Here are some questions to support children.

- How many ones are there altogether?
- Can we make an exchange? (Yes or No)
- How many do we exchange? (10 ones for 1 ten, show exchanged 10 in tens column by writing 1 in column)
- How many ones do we have left? (Write in ones column)

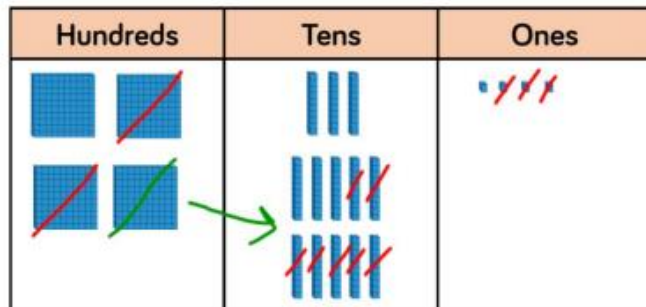
Repeat for each column.

Children in Year 2 will use Base 10 to show calculations in the horizontal format, ensuring understanding of the tens digit and the ones digit. Children in Year 3 will build on this and when ready will begin the vertical positioning using this alongside.

Base 10/Dienes (subtraction)



$$\begin{array}{r} \overset{5}{\cancel{6}} \overset{1}{5} \\ - 28 \\ \hline 37 \end{array}$$



$$\begin{array}{r} \overset{3}{\cancel{4}} \overset{1}{3} \overset{5}{5} \\ - 273 \\ \hline 162 \end{array}$$

Children in Year 2 will use Base 10 to show calculations in the horizontal format, ensuring understanding of the tens digit and the ones digit. Children in Year 3 will build on this and when ready will begin the vertical positioning using this alongside.

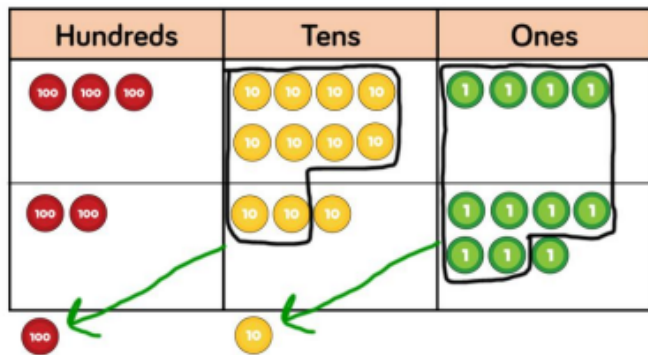
Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

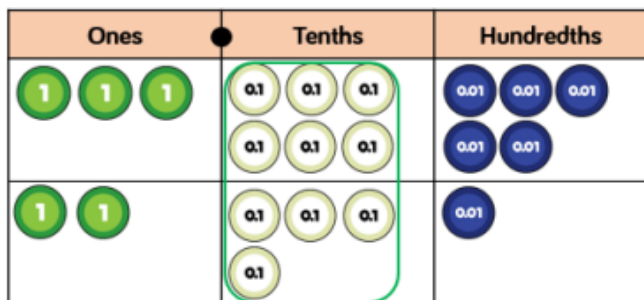
Children should first subtract without an exchange before moving on to subtraction with exchange. When building the model, children should just make the minuend using Base 10, they then subtract the subtrahend. Highlight this difference to addition to avoid errors by making both numbers. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.

This model is efficient with up to 4-digit numbers. Place value counters are more efficient with larger numbers and decimals.

Place Value Counters (addition)



$$\begin{array}{r} 384 \\ + 237 \\ \hline 621 \\ 1 \quad 1 \end{array}$$



$$\begin{array}{r} 3.65 \\ + 2.41 \\ \hline 6.06 \\ 1 \end{array}$$

Children in Year 2 will use counters to show calculations in the horizontal format, ensuring understanding of the tens digit and the ones digit. Children in Year 3 will build on this and when ready will begin the vertical positioning using this alongside.

Benefits

Using place value counters is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

Children should first add without an exchange before moving on to addition with exchange. Different place value counters can be used to represent larger numbers or decimals. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.

When adding money, children can also use coins to support their understanding. It is important that children consider how the coins link to the written calculation especially when adding decimal amounts.

Place Value Counters (Subtraction)

Hundreds	Tens	Ones

$$\begin{array}{r} 652 \\ - 207 \\ \hline 445 \end{array}$$

Thousands	Hundreds	Tens	Ones

$$\begin{array}{r} 4357 \\ - 2735 \\ \hline 1622 \end{array}$$

Children in Year 2 will use counters to show calculations in the horizontal format, ensuring understanding of the tens digit and the ones digit. Children in Year 3 will build on this and when ready will begin the vertical positioning using this alongside.

Benefits

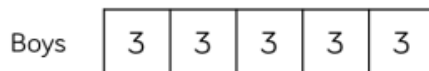
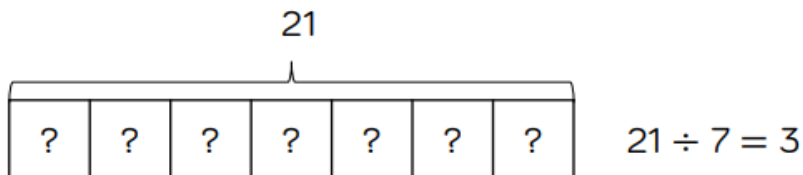
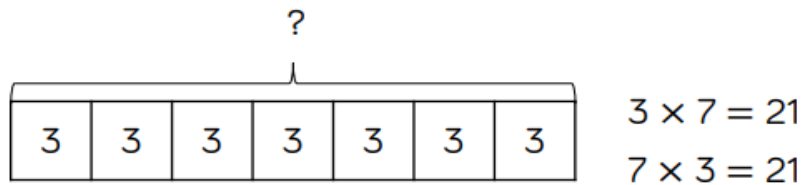
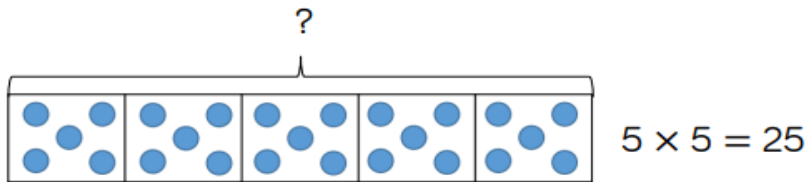
Using place value counters is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

Children should first subtract without an exchange before moving on to subtraction with exchange. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.

When building the model, children should just make the minuend using counters, they then subtract the subtrahend. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.

Multiplication and Division

Bar Model



Benefits

Children can use the single bar model to represent multiplication as repeated addition. They could use counters, cubes or dots within the bar model to support calculation before moving on to placing digits into the bar model to represent the multiplication.

Division can be represented by showing the total of the bar model and then dividing the bar model into equal groups.

It is important when solving word problems that the bar model represents the problem.

Sometimes, children may look at scaling problems. In this case, more than one bar model is useful to represent this type of problem, e.g. There are 3 girls in a group. There are 5 times more boys than girls. How many boys are there?

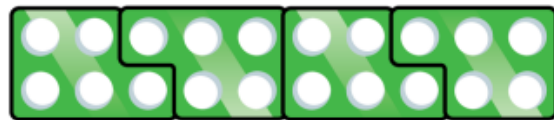
The multiple bar model provides an opportunity to compare the groups.

Number Shapes



$$5 \times 4 = 20$$

$$4 \times 5 = 20$$

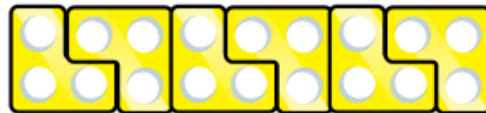


$$5 \times 4 = 20$$

$$4 \times 5 = 20$$



$$18 \div 3 = 6$$



Cuisenaire rods also support this through repeated shape patterns.

Benefits

Number shapes support children's understanding of multiplication as repeated addition.

Children can build multiplications in a row using the number shapes. When using odd numbers, encourage children to interlock the shapes so there are no gaps in the row. They can then use the tens number shapes along with other necessary shapes over the top of the row to check the total. Using the number shapes in multiplication can support children in discovering patterns of multiplication e.g. odd \times odd = even, odd \times even = odd, even \times even = even.

When dividing, number shapes support children's understanding of division as grouping. Children make the number they are dividing and then place the number shape they are dividing by over the top of the number to find how many groups of the number there are altogether e.g. There are 6 groups of 3 in 18.

Bead Strings



$$5 \times 3 = 15$$
$$3 \times 5 = 15$$

$$15 \div 3 = 5$$



$$5 \times 3 = 15$$
$$3 \times 5 = 15$$

$$15 \div 5 = 3$$



$$4 \times 5 = 20$$
$$5 \times 4 = 20$$

$$20 \div 4 = 5$$

Benefits

Bead strings to 100 can support children in their understanding of multiplication as repeated addition. Children can build the multiplication using the beads. The colour of beads supports children in seeing how many groups of 10 they have, to calculate the total more efficiently.

Encourage children to count in multiples as they build the number e.g. 4, 8, 12, 16, 20.

Children can also use the bead string to count forwards and backwards in multiples, moving the beads as they count.

When dividing, children build the number they are dividing and then group the beads into the number they are dividing by e.g. 20 divided by 4 - Make 20 and then group the beads into groups of four. Count how many groups you have made to find the answer.

Number Tracks



$$6 \times 3 = 18$$

$$3 \times 6 = 18$$



$$18 \div 3 = 6$$

This is a subtle pre-step to the number line which allows children to put counters on the number inside a box to count on or back from. It supports seeing each number in its own box.

Benefits

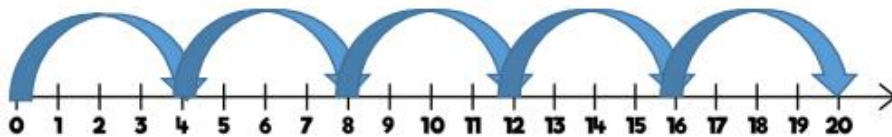
Number tracks are useful to support children to count in multiples, forwards and backwards. Moving counters or cubes along the number track can support children to keep track of their counting. Translucent counters help children to see the number they have landed on whilst counting.

When multiplying, children place their counter on 0 to start and then count on to find the product of the numbers.

When dividing, children place their counter on the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0. Children record how many jumps they have made to find the answer to the division.

Number tracks can be useful with smaller multiples but when reaching larger numbers they can become less efficient.

Number Lines (labelled)



$$4 \times 5 = 20$$

$$5 \times 4 = 20$$



$$20 \div 4 = 5$$

Can be used to show two multiplication/division come to the same answer – commutative multiplication checking. Relationship between multiplication and division.

Benefits

Labelled number lines are useful to support children to count in multiples, forwards and backwards as well as calculating single-digit multiplications.

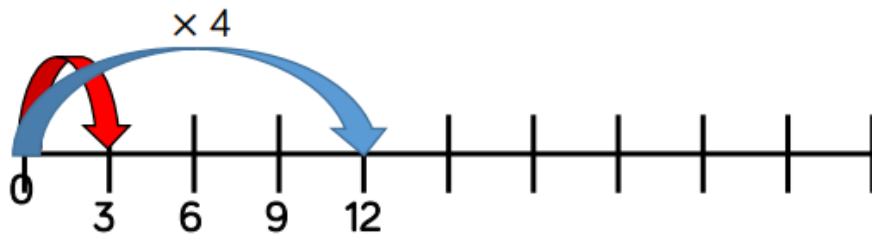
When multiplying, children start at 0 and then count on to find the product of the numbers.

When dividing, start at the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0.

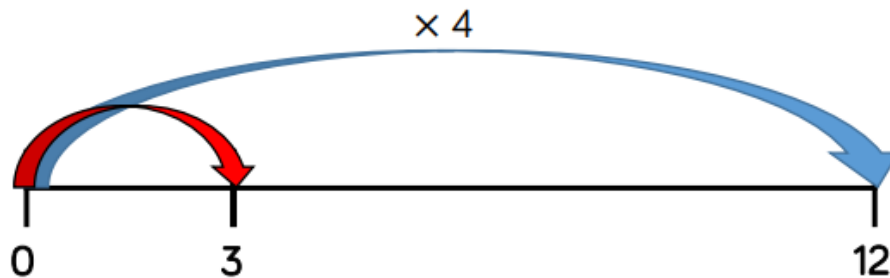
Children record how many jumps they have made to find the answer to the division.

Labelled number lines can be useful with smaller multiples, however they become inefficient as numbers become larger due to the required size of the number line.

Number Lines (blank)



A red car travels 3 miles.
A blue car 4 times further.
How far does the blue car travel?



A blue car travels 12 miles.
A red car 4 times less.
How far does the red car travel?

Benefits

Children can use blank number lines to represent scaling as multiplication or division.

Blank number lines with intervals can support children to represent scaling accurately. Children can label intervals with multiples to calculate scaling problems.

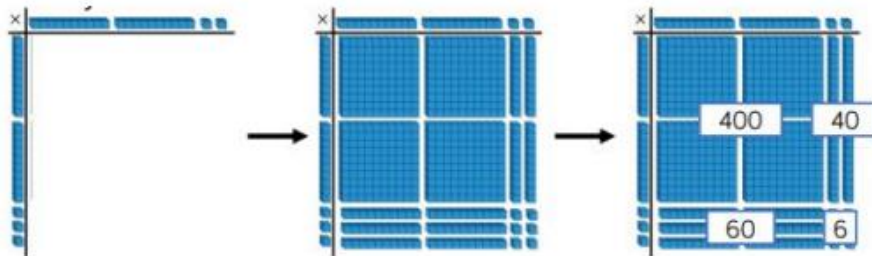
Blank number lines without intervals can also be used for children to represent scaling.

Base 10/Dienes (multiplication)

Hundreds	Tens	Ones
		●●●●
		●●●●
		●●●●

↓

$$\begin{array}{r}
 24 \\
 \times 3 \\
 \hline
 72 \\
 \hline
 1
 \end{array}$$



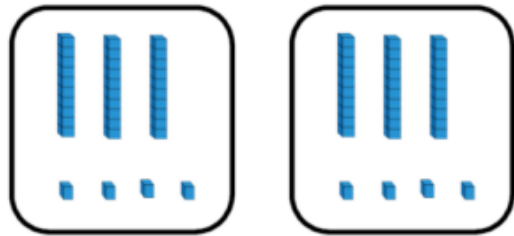
Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written representations match.

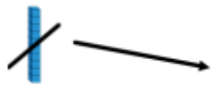
As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed.

Base 10 also supports the area model of multiplication well. Children use the equipment to build the number in a rectangular shape which they then find the area of by calculating the total value of the pieces. This area model can be linked to the grid method or the formal column method of multiplying 2-digits by 2-digits.

Base 10/Dienes (division)

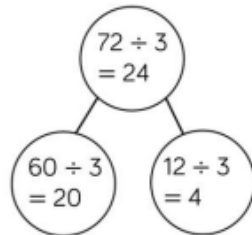


$$68 \div 2 = 34$$



Tens	Ones

$$72 \div 3 = 24$$



The expanded method for multiplication and division should always be used first to ensure understanding of the exchange.

Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of division.

When numbers become larger, it can be an effective way to move children from representing numbers as ones towards representing them as tens and ones in order to divide. Children can then share the Base 10/ Dienes between different groups e.g. by drawing circles or by rows on a place value grid.

When they are sharing, children start with the larger place value and work from left to right. If there are any left in a column, they exchange e.g. one ten for ten ones. When recording, encourage children to use the part-whole model so they can consider how the number has been partitioned in order to divide. This will support them with mental methods.

Place Value Counters (multiplication)

Hundreds	Tens	Ones
	10 10 10	1 1 1 1
	10 10 10	1 1 1 1
	10 10 10	1 1 1 1
	10 10 10	1 1 1 1
	10 10 10	1 1 1 1
100	10 10	

$$\begin{array}{r}
 34 \\
 \times 5 \\
 \hline
 170 \\
 \hline
 1\ 2
 \end{array}$$

×	10 10 10 10	1 1 1 1
10	100 100 100 100	10 10 10 10
10	100 100 100 100	10 10 10 10
10	100 100 100 100	10 10 10 10
1	10 10 10 10	1 1 1 1
1	10 10 10 10	1 1 1 1

$$\begin{array}{r}
 44 \\
 \times 32 \\
 \hline
 8 \\
 80 \\
 120 \\
 + 1200 \\
 \hline
 1408
 \end{array}$$

Benefits

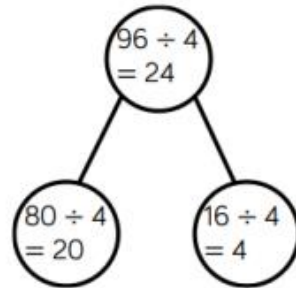
Using place value counters is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written match.

As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed. The counters should be used to support the understanding of the written method rather than support the arithmetic.

Place value counters also support the area model of multiplication well. Children can see how to multiply 2-digit numbers by 2-digit numbers.

Place Value Counters (division)

Tens	Ones
10 10	1 1 1 1
10 10	1 1 1 1
10 10	1 1 1 1
10 10	1 1 1 1



Thousands	Hundreds	Tens	Ones
1000 1000 1000 1000	100 100 100 100 100 100	10 10 10 10 10 10 10 10	1 1 1 1 1 1 1 1 1 1

$$4 \overline{) 1223} \\ \underline{489} \\ 12$$

Benefits

Using place value counters is an effective way to support children's understanding of division.

When working with smaller numbers, children can use place value counters to share between groups. They start by sharing the larger place value column and work from left to right. If there are any counters left over once they have been shared, they exchange the counter e.g. exchange one ten for ten ones. This method can be linked to the part-whole model to support children to show their thinking.

Place value counters also support children's understanding of short division by grouping the counters rather than sharing them. Children work from left to right through the place value columns and group the counters in the number they are dividing by. If there are any counters left over after they have been grouped, they exchange the counter e.g. exchange one hundred for ten tens.



Progression in Maths
Years N - 6

Whole school progression of skills in maths:

- Place value
- Addition and subtraction
- Multiplication and division
- Fraction, decimals and percentages
- Ratio and proportion
- Algebra
- Geometry
- Statistics

EYFS

Range 4 (Nursery) Comparison

- Beginning to compare and recognise changes in numbers of things, using words like more, lots or same

Counting

- Begins to say numbers in order, some of which are in the right order (ordinality)

Cardinality (How many?)

- In everyday situations, takes or gives two or three objects from a group
- Beginning to notice numerals (number symbols)
- Beginning to count on their fingers.

Range 5 Comparison

- Compares two small groups of up to five objects, saying when there are the same number of objects in each group, e.g. You've got two, I've got two. Same!

Counting

- May enjoy counting verbally as far as they can go
- Points or touches (tags) each item, saying one number for each item, using the stable order of 1,2,3,4,5.
- Uses some number names and number language within play, and may show fascination with large numbers
- Begin to recognise numerals 0 to 10

Cardinality

- Subitises one, two and three objects (without counting)
- Counts up to five items, recognising that the last number said represents the total counted so far (cardinal principle)
- Links numerals with amounts up to 5 and maybe beyond
- Explores using a range of their own marks and signs to which they ascribe mathematical meanings

Range 6 Comparison

- Uses number names and symbols when comparing numbers, showing interest in large numbers
- Estimates of numbers of things, showing understanding of relative size

Counting

Enjoys reciting numbers from 0 to 10 (and beyond) and back from 10 to 0

- Increasingly confident at putting numerals in order 0 to 10 (ordinality)

Cardinality

Primary Progression – Place Value

- Engages in subitising numbers to four and maybe five
- Counts out up to 10 objects from a larger group.
- Matches the numeral with a group of items to show how many there are (up to 10)

Early Learning Goals

- Have a deep understanding of number to 10, including composition of each number
- Subitise (recognise quantities without counting) up to 5
- Verbally count beyond 20, recognising the pattern of the counting system
- Compare quantities up to 10 in different contexts, recognising when one quantity is greater than, less than or the same as the other quantity
- Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Place Value: Counting	<ul style="list-style-type: none"> count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number Count numbers to 100 in numerals; count in multiples of twos, fives and tens <p>Autumn 1 Autumn 4 Spring 2 Summer 4</p>	<ul style="list-style-type: none"> count in steps of 2, 3, and 5 from 0, and in tens from any number, forward and backward <p>Autumn 1</p>	<ul style="list-style-type: none"> count from 0 in multiples of 4, 8, 50 and 100; find 10 or 100 more or less than a given number <p>Autumn 1 Autumn 3</p>	<ul style="list-style-type: none"> count in multiples of 6, 7, 9, 25 and 1000 count backwards through zero to include negative numbers <p>Autumn 1 Autumn 4</p>	<ul style="list-style-type: none"> count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000 count forwards and backwards with positive and negative whole numbers, including through zero <p>Autumn 1</p>	

<p style="writing-mode: vertical-rl; transform: rotate(180deg);">Place Value: Represent</p>	<ul style="list-style-type: none"> • identify and represent numbers using objects and pictorial representations • read and write numbers to 100 in numerals • read and write numbers from 1 to 20 in numerals and words. <p style="text-align: center;">Autumn 1 Autumn 4 Spring 2 Summer 4</p>	<ul style="list-style-type: none"> • read and write numbers to at least 100 in numerals and in words • identify, represent and estimate numbers using different representations, including the number line <p style="text-align: center;">Autumn 1</p>	<ul style="list-style-type: none"> • identify, represent and estimate numbers using different representations • read and write numbers up to 1000 in numerals and in words <p style="text-align: center;">Autumn 1</p>	<ul style="list-style-type: none"> • identify, represent and estimate numbers using different representations • read Roman numerals to 100 (I to C) and know that over time, the numeral system changed to include the concept of zero and place value <p style="text-align: center;">Autumn 1</p>	<ul style="list-style-type: none"> • read, write, (order and compare) numbers to at least 1 000 000 and determine the value of each digit • read Roman numerals to 1000 (M) and recognise years written in Roman numerals. <p style="text-align: center;">Autumn 1</p>	<ul style="list-style-type: none"> • read, write, (order and compare) numbers up to 10 000 000 and determine the value of each digit <p style="text-align: center;">Autumn 1</p>
--	--	--	--	--	--	---

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Place Value : Use PV and Compare	<ul style="list-style-type: none"> given a number, identify one more and one less <p>Autumn 1 Autumn 4 Spring 2 Summer 4</p>	<ul style="list-style-type: none"> recognise the place value of each digit in a two-digit number (tens, ones) compare and order numbers from 0 up to 100; use <, > and = signs <p>Autumn 1</p>	<ul style="list-style-type: none"> recognise the place value of each digit in a three-digit number (hundreds, tens, ones) compare and order numbers up to 1000 <p>Autumn 1</p>	<ul style="list-style-type: none"> find 1000 more or less than a given number recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones) order and compare numbers beyond 1000 <p>Autumn 1</p>	<ul style="list-style-type: none"> (read, write) order and compare numbers to at least 1 000 000 and determine the value of each digit <p>Autumn 1</p>	<ul style="list-style-type: none"> (read, write), order and compare numbers up to 10 000 000 and determine the value of each digit <p>Autumn 1</p>
Place Value: Problems & Rounding		<ul style="list-style-type: none"> use place value and number facts to solve problems. <p>Autumn 1</p>	<ul style="list-style-type: none"> solve number problems and practical problems involving these ideas <p>Autumn 1</p>	<ul style="list-style-type: none"> round any number to the nearest 10, 100 or 1000 solve number and practical problems that involve all of the above and with increasingly large positive numbers <p>Autumn 1</p>	<ul style="list-style-type: none"> interpret negative numbers in context round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000 solve number problems and practical problems that involve all of the above <p>Autumn 1</p>	<ul style="list-style-type: none"> round any whole number to a required degree of accuracy use negative numbers in context, and calculate intervals across zero solve number and practical problems that involve all of the above <p>Autumn 1</p>

EYFS:

Range 5 Composition

- Through play and exploration, beginning to learn that numbers are made up (composed) of smaller numbers
 - Beginning to use understanding of number to solve practical problems in play and meaningful activities
 - Beginning to recognise that each counting number is one more than the one before
- Separates a group of three or four objects in different ways, beginning to recognise that the total is still the same

Range 6 Composition

- Shows awareness that numbers are made up (composed) of smaller numbers, exploring partitioning in different ways with a wide range of objects
- Begins to conceptually subitise larger numbers by subitising smaller groups within the number, e.g. sees six raisins on a plate as three and three
- In practical activities, adds one and subtracts one with numbers to 10
- Begins to explore and work out mathematical problems, using signs and strategies of their own choice, including (when appropriate) standard numerals, tallies and “+” or “-”

Early Learning Goals

Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (including subtraction facts) and some number bonds to 10, including double facts

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition & Subtraction: Recall, Represent, Use	<ul style="list-style-type: none"> read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs represent and use number bonds and related subtraction facts within 20 <p style="text-align: center;">Autumn 2 Spring 1</p>	<ul style="list-style-type: none"> recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100 show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems <p style="text-align: center;">Autumn 2</p>	<ul style="list-style-type: none"> estimate the answer to a calculation and use inverse operations to check answers <p style="text-align: center;">Autumn 2</p>	<ul style="list-style-type: none"> estimate and use inverse operations to check answers to a calculation <p style="text-align: center;">Autumn 2</p>	<ul style="list-style-type: none"> use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy <p style="text-align: center;">Autumn 2</p>	

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition & Subtraction: Calculations	<ul style="list-style-type: none"> add and subtract one-digit and two-digit numbers to 20, including zero <p style="text-align: center;">Autumn 2 Spring 1</p>	<ul style="list-style-type: none"> add and subtract numbers using concrete objects, pictorial representations, and mentally, including: <ul style="list-style-type: none"> a two-digit number and ones a two-digit number and tens two two-digit numbers adding three one-digit numbers <p style="text-align: center;">Autumn 2</p>	<ul style="list-style-type: none"> add and subtract numbers mentally, including: <ul style="list-style-type: none"> a three-digit number and ones a three-digit number and tens a three-digit number and hundreds add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction <p style="text-align: center;">Autumn 2</p>	<ul style="list-style-type: none"> add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate <p style="text-align: center;">Autumn 2</p>	<ul style="list-style-type: none"> add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction) add and subtract numbers mentally with increasingly large numbers <p style="text-align: center;">Autumn 2</p>	<ul style="list-style-type: none"> perform mental calculations, including with mixed operations and large numbers use their knowledge of the order of operations to carry out calculations involving the four operations <p style="text-align: center;">Autumn 2</p>

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition & Subtraction: Solve Problems	<ul style="list-style-type: none"> • solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as $7 = \square - 9$ 	<ul style="list-style-type: none"> • solve problems with addition and subtraction: <ul style="list-style-type: none"> ➤ using concrete objects and pictorial representations, including those involving numbers, quantities and measures ➤ applying their increasing knowledge of mental and written methods 	<ul style="list-style-type: none"> • solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction 	<ul style="list-style-type: none"> • solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why 	<ul style="list-style-type: none"> • solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why • solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign 	<ul style="list-style-type: none"> • solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
	<p style="text-align: center;">Autumn 2 Spring 1</p>	<p style="text-align: center;">Autumn 2</p>	<p style="text-align: center;">Autumn 2</p>	<p style="text-align: center;">Autumn 2</p>	<p style="text-align: center;">Autumn 2</p>	<p style="text-align: center;">Autumn 2</p>

Early Learning Goals

Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Multiplication & Division: Recall, Represent, Use		<ul style="list-style-type: none"> recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot <p style="text-align: center;">Autumn 4 Spring 1</p>	<ul style="list-style-type: none"> recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables <p style="text-align: center;">Autumn 3</p>	<ul style="list-style-type: none"> recall multiplication and division facts for multiplication tables up to 12×12 use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers recognise and use factor pairs and commutativity in mental calculations <p style="text-align: center;">Autumn 4 Spring 1</p>	<ul style="list-style-type: none"> identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers establish whether a number up to 100 is prime and recall prime numbers up to 19 recognise and use square numbers and cube numbers, and the notation for squared (²) and cubed (³) <p style="text-align: center;">Autumn 4</p>	<ul style="list-style-type: none"> identify common factors, common multiples and prime numbers use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy. <p style="text-align: center;">Autumn 2</p>

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Multiplication & Division: Calculations		<ul style="list-style-type: none"> calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (\times), division (\div) and equals ($=$) signs 	<ul style="list-style-type: none"> write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods 	<ul style="list-style-type: none"> multiply two-digit and three-digit numbers by a one-digit number using formal written layout 	<ul style="list-style-type: none"> multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers multiply and divide numbers mentally drawing upon known facts divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context multiply and divide whole numbers and those involving decimals by 10, 100 and 1000 	<ul style="list-style-type: none"> multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context perform mental calculations, including with mixed operations and large numbers
		<p style="text-align: center;">Autumn 4 Spring 1</p>	<p style="text-align: center;">Autumn 3 Spring 1</p>	<p style="text-align: center;">Spring 1</p>	<p style="text-align: center;">Autumn 4 Spring 1 Summer 1</p>	<p style="text-align: center;">Autumn 2</p>

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Multiplication & Division: Solve Problems	<ul style="list-style-type: none"> solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher <p style="text-align: center;">Summer 1</p>	<ul style="list-style-type: none"> solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts <p style="text-align: center;">Autumn 4 Spring 1</p>	<ul style="list-style-type: none"> solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects <p style="text-align: center;">Spring 1</p>	<ul style="list-style-type: none"> solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects <p style="text-align: center;">Spring 1</p>	<ul style="list-style-type: none"> solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates <p style="text-align: center;">Autumn 4 Spring 1</p>	<ul style="list-style-type: none"> solve problems involving addition, subtraction, multiplication and division <p style="text-align: center;">Autumn 2</p>
Multiplication & Division: Combined Operations					<ul style="list-style-type: none"> solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign <p style="text-align: center;">Spring 1</p>	<ul style="list-style-type: none"> use their knowledge of the order of operations to carry out calculations involving the four operations <p style="text-align: center;">Autumn 2</p>

--	--	--	--	--	--	--

Early Learning Goals

Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Fractions: Recognise and Write	<ul style="list-style-type: none"> recognise, find and name a half as one of two equal parts of an object, shape or quantity recognise, find and name a quarter as one of four equal parts of an object, shape or quantity <p style="text-align: center;">Summer 2</p>	<ul style="list-style-type: none"> recognise, find, name and write fractions $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{4}$ and $\frac{3}{4}$ of a length, shape, set of objects or quantity <p style="text-align: center;">Spring 4</p>	<ul style="list-style-type: none"> count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10 recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators recognise and use fractions as numbers: unit fractions and non-unit fractions with small denominators <p style="text-align: center;">Spring 5</p>	<ul style="list-style-type: none"> count up and down in hundredths; recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten. <p style="text-align: center;">Spring 3</p>	<ul style="list-style-type: none"> identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number [for example, $\frac{2}{5} + \frac{4}{5} = \frac{6}{5} = 1\frac{1}{5}$] <p style="text-align: center;">Spring 2</p>	
Fractions: Compare		<ul style="list-style-type: none"> Recognise the equivalence of $\frac{2}{4}$ and $\frac{1}{2}$ <p style="text-align: center;">Spring 4</p>	<ul style="list-style-type: none"> recognise and show, using diagrams, equivalent fractions with small denominators compare and order unit fractions, and fractions with the same denominators <p style="text-align: center;">Summer 1</p>	<ul style="list-style-type: none"> recognise and show, using diagrams, families of common equivalent fractions <p style="text-align: center;">Spring 3</p>	<ul style="list-style-type: none"> compare and order fractions whose denominators are all multiples of the same number <p style="text-align: center;">Spring 2</p>	<ul style="list-style-type: none"> use common factors to simplify fractions; use common multiples to express fractions in the same denomination compare and order fractions, including fractions > 1 <p style="text-align: center;">Autumn 3</p>

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Fractions: Calculations		<ul style="list-style-type: none"> write simple fractions for example, $\frac{1}{2}$ of 6 = 3 <p style="text-align: center;">Spring 4</p>	<ul style="list-style-type: none"> add and subtract fractions with the same denominator within one whole [for example, $\frac{5}{7} + \frac{1}{7} = \frac{6}{7}$] <p style="text-align: center;">Summer 1</p>	<ul style="list-style-type: none"> add and subtract fractions with the same denominator <p style="text-align: center;">Spring 3</p>	<ul style="list-style-type: none"> add and subtract fractions with the same denominator and denominators that are multiples of the same number multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams <p style="text-align: center;">Spring 3</p>	<ul style="list-style-type: none"> add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions multiply simple pairs of proper fractions, writing the answer in its simplest form [for example, $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$] divide proper fractions by whole numbers [for example, $\frac{1}{3} \div 2 = \frac{1}{6}$] <p style="text-align: center;">Autumn 3</p>
Fractions: Solve Problems			<ul style="list-style-type: none"> solve problems that involve all of the above <p style="text-align: center;">Spring 5 Summer 1</p>	<ul style="list-style-type: none"> solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number <p style="text-align: center;">Spring 3</p>		

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Decimals: Recognise and Write				<ul style="list-style-type: none"> recognise and write decimal equivalents of any number of tenths or hundredths recognise and write decimal equivalents to $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}$ <p style="text-align: center;">Spring 4 Summer 1</p>	<ul style="list-style-type: none"> read and write decimal numbers as fractions [for example, $0.71 = \frac{71}{100}$] recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents <p style="text-align: center;">Spring 3</p>	<ul style="list-style-type: none"> identify the value of each digit in numbers given to three decimal places <p style="text-align: center;">Spring 1</p>
Decimals: Compare				<ul style="list-style-type: none"> round decimals with one decimal place to the nearest whole number compare numbers with the same number of decimal places up to two decimal places <p style="text-align: center;">Summer 1</p>	<ul style="list-style-type: none"> round decimals with two decimal places to the nearest whole number and to one decimal place read, write, order and compare numbers with up to three decimal places <p style="text-align: center;">Spring 3</p>	

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Decimals: Calculations & Problems				<ul style="list-style-type: none"> find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths <p style="text-align: center;">Spring 4</p>	<ul style="list-style-type: none"> solve problems involving number up to three decimal places <p style="text-align: center;">Summer 1</p>	<ul style="list-style-type: none"> multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places multiply one-digit numbers with up to two decimal places by whole numbers use written division methods in cases where the answer has up to two decimal places solve problems which require answers to be rounded to specified degrees of accuracy <p style="text-align: center;">Spring 1</p>

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Fractions, Decimals and Percentages				<ul style="list-style-type: none"> solve simple measure and money problems involving fractions and decimals to two decimal places <p>Spring 3 Spring 4 Summer 1</p>	<ul style="list-style-type: none"> recognise the per cent symbol (%) and understand that per cent relates to 'number of parts per hundred', and write percentages as a fraction with denominator 100, and as a decimal solve problems which require knowing percentage and decimal equivalents of $\frac{1}{2}, \frac{1}{4}, \frac{1}{5}, \frac{2}{5}, \frac{4}{5}$ and those fractions with a denominator of a multiple of 10 or 25 <p>Spring 3</p>	<ul style="list-style-type: none"> associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375] for a simple fraction [for example, $\frac{3}{8}$] recall and use equivalences between simple fractions, decimals and percentages, including in different contexts <p>Spring 1 Spring 2</p>

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Ratio and Proportion						<ul style="list-style-type: none"> • solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts • solve problems involving the calculation of percentages [for example, of measures, and such as 15% of 360] and the use of percentages for comparison • solve problems involving similar shapes where the scale factor is known or can be found • solve problems involving unequal sharing and grouping using knowledge of fractions and multiples. <p style="text-align: right;">Spring 6</p>

Primary Progression – Algebra

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Algebra	<ul style="list-style-type: none"> solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as $7 = \square - 9$ 	<ul style="list-style-type: none"> recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems 	<ul style="list-style-type: none"> solve problems, including missing number problems 			<ul style="list-style-type: none"> use simple formulae generate and describe linear number sequences express missing number problems algebraically find pairs of numbers that satisfy an equation with two unknowns enumerate possibilities of combinations of two variables. <p style="text-align: right;">Spring 3</p>

Note – although algebraic notation is not introduced until Y6, algebraic thinking starts much earlier as exemplified by the ‘missing number’ objectives from Y1/2/3

EYFS

Range 4 (Nursery)

- Explores differences in size, length, weight and capacity
- Beginning to understand some talk about immediate past and future
- Beginning to anticipate times of the day such as mealtimes or home time

Range 5

- In meaningful contexts, finds the longer or shorter, heavier or lighter and more/less full of two items
- Recalls a sequence of events in everyday life and stories

Range 6

- Enjoys tackling problems involving prediction and discussion of comparisons of length, weight or capacity, paying attention to fairness and accuracy
- Becomes familiar with measuring tools in everyday experiences and play
- Is increasingly able to order and sequence events using everyday language related to time
- Beginning to experience measuring time with timers and calendars

Although shape, space and measure have been removed from the ELGs, planning should ensure that ‘the curriculum includes rich opportunities for children to develop their spatial reasoning skills across all areas of mathematics including shape, space and measure.

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Measurement: Using Measures	<ul style="list-style-type: none"> • compare, describe and solve practical problems for: <ul style="list-style-type: none"> ➤ lengths and heights [for example, long/short, longer/shorter, tall/short, double/half] ➤ mass/weight [for example, heavy/light, heavier than, lighter than] ➤ capacity and volume [for example, full/empty, more than, less than, half, half full, quarter] ➤ time [for example, quicker, slower, earlier, later] • measure and begin to record the following: <ul style="list-style-type: none"> ➤ lengths and heights ➤ mass/weight ➤ capacity and volume ➤ time (hours, minutes, seconds) 	<ul style="list-style-type: none"> • choose and use appropriate standard units to estimate and measure length/height in any direction (m/cm); mass (kg/g); temperature (°C); capacity (litres/ml) to the nearest appropriate unit, using rulers, scales, thermometers and measuring vessels • compare and order lengths, mass, volume/capacity and record the results using >, < and = 	<ul style="list-style-type: none"> • measure, compare, add and subtract: lengths (m/cm/mm); mass (kg/g); volume/capacity (l/ml) 	<ul style="list-style-type: none"> • Convert between different units of measure [for example, kilometre to metre; hour to minute] • estimate, compare and calculate different measures 	<ul style="list-style-type: none"> • convert between different units of metric measure (for example, kilometre and metre; centimetre and metre; centimetre and millimetre; gram and kilogram; litre and millilitre) • understand and use approximate equivalences between metric units and common imperial units such as inches, pounds and pints • use all four operations to solve problems involving measure [for example, length, mass, volume, money] using decimal notation, including scaling 	<ul style="list-style-type: none"> • solve problems involving the calculation and conversion of units of measure, using decimal notation up to three decimal places where appropriate • use, read, write and convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to a larger unit, and vice versa, using decimal notation to up to three decimal places • convert between miles and kilometres
	Spring 3 Spring 4 Summer 6	Spring 5 Summer 4	Spring 4 Summer 4	Autumn 3 Spring 2 Summer 3	Summer 1 Summer 4 Summer 5	Spring 4

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Measurement: Money	<ul style="list-style-type: none"> recognise and know the value of different denominations of coins and notes <p>Summer 5</p>	<ul style="list-style-type: none"> recognise and use symbols for pounds (£) and pence (p); combine amounts to make a particular value find different combinations of coins that equal the same amounts of money solve simple problems in a practical context involving addition and subtraction of money of the same unit, including giving change <p>Autumn 3</p>	<ul style="list-style-type: none"> add and subtract amounts of money to give change, using both £ and p in practical contexts <p>Spring 2</p>	<ul style="list-style-type: none"> estimate, compare and calculate different measures, including money in pounds and pence <p>Summer 2</p>	<ul style="list-style-type: none"> use all four operations to solve problems involving measure [for example, money] <p>Summer 1</p>	

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Measurement: Time	<ul style="list-style-type: none"> sequence events in chronological order using language [for example, before and after, next, first, today, yesterday, tomorrow, morning, afternoon and evening] recognise and use language relating to dates, including days of the week, weeks, months and years tell the time to the hour and half past the hour and draw the hands on a clock face to show these times 	<ul style="list-style-type: none"> compare and sequence intervals of time tell and write the time to five minutes, including quarter past/to the hour and draw the hands on a clock face to show these times know the number of minutes in an hour and the number of hours in a day 	<ul style="list-style-type: none"> tell and write the time from an analogue clock, including using Roman numerals from I to XII, and 12-hour and 24-hour clocks estimate and read time with increasing accuracy to the nearest minute; record and compare time in terms of seconds, minutes and hours; use vocabulary such as o'clock, a.m./p.m., morning, afternoon, noon and midnight know the number of seconds in a minute and the number of days in each month, year and leap year compare durations of events [for example to calculate the time taken by particular events or tasks] 	<ul style="list-style-type: none"> read, write and convert time between analogue and digital 12- and 24-hour clocks solve problems involving converting from hours to minutes; minutes to seconds; years to months; weeks to days 	<ul style="list-style-type: none"> solve problems involving converting between units of time 	<ul style="list-style-type: none"> use, read, write and convert between standard units, converting measurements of time from a smaller unit of measure to a larger unit, and vice versa
	Summer 6	Summer 3	Summer 2	Summer 3	Summer 4	Year 5 Summer 4

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Measurement: Perimeter, Area, Volume			<ul style="list-style-type: none"> measure the perimeter of simple 2-D shapes <p style="text-align: center;">Spring 4</p>	<ul style="list-style-type: none"> measure and calculate the perimeter of a rectilinear figure (including squares) in centimetres and metres find the area of rectilinear shapes by counting squares <p style="text-align: center;">Autumn 3 Spring 2</p>	<ul style="list-style-type: none"> measure and calculate the perimeter of composite rectilinear shapes in centimetres and metres calculate and compare the area of rectangles (including squares), and including using standard units, square centimetres (cm²) and square metres (m²) and estimate the area of irregular shapes estimate volume [for example, using 1 cm³ blocks to build cuboids (including cubes)] and capacity [for example, using water] <p style="text-align: center;">Autumn 5 Summer 5</p>	<ul style="list-style-type: none"> recognise that shapes with the same areas can have different perimeters and vice versa recognise when it is possible to use formulae for area and volume of shapes calculate the area of parallelograms and triangles calculate, estimate and compare volume of cubes and cuboids using standard units, including cubic centimetres (cm³) and cubic metres (m³), and extending to other units [for example, mm³ and km³] <p style="text-align: center;">Spring 5</p>

EYFS

Range 4 (Nursery) Spatial Awareness

- Moves their bodies and toys around objects and explores fitting into spaces
- Begins to remember their way around familiar environments
- Responds to some spatial and positional language
- Explores how things look from different viewpoints including things that are near or far away

Shape

- Chooses puzzle pieces and tries to fit them in
- Recognises that two objects have the same shape
- Makes simple constructions

Pattern

- Joins in and anticipates repeated sound and action patterns
- Is interested in what happens next using the pattern of everyday routines

Range 5

Spatial Awareness

- Responds to and uses language of position and direction
- Predicts, moves and rotates objects to fit the space or create the shape they would like

Shape

- Chooses items based on their shape which are appropriate for the child's purpose
- Responds to both informal language and common shape names
- Shows awareness of shape similarities and differences between objects
- Enjoys partitioning and combining shapes to make new shapes with 2D and 3D shapes
- Attempts to create arches and enclosures when building, using trial and improvement to select blocks

Pattern

- Creates their own spatial patterns showing some organisation or regularity

- Explores and adds to simple linear patterns of two or three repeating items, e.g. stick, leaf (AB) or stick, leaf, stone (ABC)
- Joins in with simple patterns in sounds, objects, games and stories dance and movement, predicting what comes next

Range 6

Spatial Awareness

- Uses spatial language, including following and giving directions, using relative terms and describing what they see from different viewpoints
- Investigates turning and flipping objects in order to make shapes fit and create models; predicting and visualising how they will look (spatial reasoning)
- May enjoy making simple maps of familiar and imaginative environments, with landmarks

Shape

- Uses informal language and analogies, (e.g. heart-shaped and hand-shaped leaves), as well as mathematical terms to describe shapes
- Enjoys composing and decomposing shapes, learning which shapes combine to make other shapes
- Uses own ideas to make models of increasing complexity, selecting blocks needed, solving problems and visualising what they will build

Pattern

- Spots patterns in the environment, beginning to identify the pattern “rule”
- Chooses familiar objects to create and recreate repeating patterns beyond AB patterns and begins to identify the unit of repeat

Although shape, space and measure have been removed from the ELGs, planning should ensure that ‘the curriculum includes rich opportunities for children to develop their spatial reasoning skills across all areas of mathematics including shape, space and measure.

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Geometry: 2-D Shapes	<ul style="list-style-type: none"> recognise and name common 2-D shapes [for example, rectangles (including squares), circles and triangles] <p style="text-align: center;">Autumn 3</p>	<ul style="list-style-type: none"> identify and describe the properties of 2-D shapes, including the number of sides and line symmetry in a vertical line identify 2-D shapes on the surface of 3-D shapes, [for example, a circle on a cylinder and a triangle on a pyramid] compare and sort common 2-D shapes and everyday objects <p style="text-align: center;">Spring 3</p>	<ul style="list-style-type: none"> draw 2-D shapes <p style="text-align: center;">Summer 3</p>	<ul style="list-style-type: none"> compare and classify geometric shapes, including quadrilaterals and triangles, based on their properties and sizes identify lines of symmetry in 2-D shapes presented in different orientations <p style="text-align: center;">Summer 5</p>	<ul style="list-style-type: none"> distinguish between regular and irregular polygons based on reasoning about equal sides and angles. use the properties of rectangles to deduce related facts and find missing lengths and angles <p style="text-align: center;">Summer 2</p>	<ul style="list-style-type: none"> draw 2-D shapes using given dimensions and angles compare and classify geometric shapes based on their properties and sizes illustrate and name parts of circles, including radius, diameter and circumference and know that the diameter is twice the radius <p style="text-align: center;">Summer 1</p>
Geometry: 3-D Shapes	<ul style="list-style-type: none"> recognise and name common 3-D shapes [for example, cuboids (including cubes), pyramids and spheres] <p style="text-align: center;">Autumn 3</p>	<ul style="list-style-type: none"> recognise and name common 3-D shapes [for example, cuboids (including cubes), pyramids and spheres]. compare and sort common 3-D shapes and everyday objects <p style="text-align: center;">Spring 3</p>	<ul style="list-style-type: none"> make 3-D shapes using modelling materials; recognise 3-D shapes in different orientations and describe them <p style="text-align: center;">Summer 3</p>		<ul style="list-style-type: none"> identify 3-D shapes, including cubes and other cuboids, from 2-D representations <p style="text-align: center;">Summer 2</p>	<ul style="list-style-type: none"> recognise, describe and build simple 3-D shapes, including making nets <p style="text-align: center;">Summer 1</p>

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Geometry: Angles & Lines			<ul style="list-style-type: none"> recognise angles as a property of shape or a description of a turn identify right angles, recognise that two right angles make a half-turn, three make three quarters of a turn and four a complete turn; identify whether angles are greater than or less than a right angle identify horizontal and vertical lines and pairs of perpendicular and parallel lines 	<ul style="list-style-type: none"> identify acute and obtuse angles and compare and order angles up to two right angles by size identify lines of symmetry in 2-D shapes presented in different orientations complete a simple symmetric figure with respect to a specific line of symmetry 	<ul style="list-style-type: none"> know angles are measured in degrees: estimate and compare acute, obtuse and reflex angles draw given angles, and measure them in degrees identify: <ul style="list-style-type: none"> angles at a point and one whole turn (total 360°) angles at a point on a straight line and $\frac{1}{2}$ a turn (total 180°) other multiples of 90° 	<ul style="list-style-type: none"> find unknown angles in any triangles, quadrilaterals, and regular polygons recognise angles where they meet at a point, are on a straight line, or are vertically opposite, and find missing angles
			Summer 3	Summer 5	Summer 2	Summer 1

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Geometry: Position & Direction	<ul style="list-style-type: none"> describe position, direction and movement, including whole, half, quarter and three-quarter turns <p style="text-align: center;">Summer 3</p>	<ul style="list-style-type: none"> order and arrange combinations of mathematical objects in patterns and sequences use mathematical vocabulary to describe position, direction and movement, including movement in a straight line and distinguishing between rotation as a turn and in terms of right angles for quarter, half and three-quarter turns (clockwise and anti-clockwise) <p style="text-align: center;">Spring 3 Summer 1</p>		<ul style="list-style-type: none"> describe positions on a 2-D grid as coordinates in the first quadrant describe movements between positions as translations of a given unit to the left/right and up/down plot specified points and draw sides to complete a given polygon <p style="text-align: center;">Summer 6</p>	<ul style="list-style-type: none"> identify, describe and represent the position of a shape following a reflection or translation, using the appropriate language, and know that the shape has not changed <p style="text-align: center;">Summer 3</p>	<ul style="list-style-type: none"> describe positions on the full coordinate grid (all four quadrants) draw and translate simple shapes on the coordinate plane, and reflect them in the axes <p style="text-align: center;">Autumn 4</p>

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Statistics: Present and Interpret		<ul style="list-style-type: none"> interpret and construct simple pictograms, tally charts, block diagrams and simple tables <p style="text-align: center;">Spring 2</p>	<ul style="list-style-type: none"> interpret and present data using bar charts, pictograms and tables <p style="text-align: center;">Spring 3</p>	<ul style="list-style-type: none"> interpret and present discrete and continuous data using appropriate graphical methods, including bar charts and time graphs <p style="text-align: center;">Summer 4</p>	<ul style="list-style-type: none"> complete, read and interpret information in tables, including timetables <p style="text-align: center;">Autumn 3</p>	<ul style="list-style-type: none"> interpret and construct pie charts and line graphs and use these to solve problems <p style="text-align: center;">Summer 3</p>
Statistics: Solve Problems		<ul style="list-style-type: none"> ask and answer simple questions by counting the number of objects in each category and sorting the categories by quantity ask and answer questions about totalling and comparing categorical data <p style="text-align: center;">Spring 2</p>	<ul style="list-style-type: none"> solve one-step and two-step questions [for example, 'How many more?' and 'How many fewer?'] using information presented in scaled bar charts and pictograms and tables <p style="text-align: center;">Spring 3</p>	<ul style="list-style-type: none"> solve comparison, sum and difference problems using information presented in bar charts, pictograms, tables and other graphs <p style="text-align: center;">Summer 4</p>	<ul style="list-style-type: none"> solve comparison, sum and difference problems using information presented in a line graph <p style="text-align: center;">Autumn 3</p>	<ul style="list-style-type: none"> calculate and interpret the mean as an average <p style="text-align: center;">Summer 3</p>

End of year expectations

Year 1	Year 1	Year 1	Year 1	Year 1	Year 1
<ul style="list-style-type: none"> count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number Count numbers to 100 in numerals; count in multiples of twos, fives and tens 	<ul style="list-style-type: none"> identify and represent numbers using objects and pictorial representations read and write numbers to 100 in numerals read and write numbers from 1 to 20 in numerals and words 	<ul style="list-style-type: none"> given a number, identify one more and one less 	<ul style="list-style-type: none"> recognise, find and name a half as one of two equal parts of an object, shape or quantity recognise, find and name a quarter as one of four equal parts of an object, shape or quantity 	<ul style="list-style-type: none"> add and subtract one-digit and two-digit numbers to 20, including zero 	<ul style="list-style-type: none"> solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher
Autumn 1 Spring 1 Spring 3 Summer 4	Autumn 1 Spring 1 Spring 3 Summer 4	Autumn 1 Spring 1 Spring 3 Summer 4	Summer 2	Autumn 2 Spring 2	Summer 1

Year 1	Year 1	Year 1	Year 1	Year 1	Year 1	Year 1
<ul style="list-style-type: none"> solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as $7 = \square - 9$ 	<ul style="list-style-type: none"> <i>solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as $7 = \square - 9$</i> 	<ul style="list-style-type: none"> compare, describe and solve practical problems for: <ul style="list-style-type: none"> lengths and heights mass/weight capacity and volume time measure and begin to record the following: <ul style="list-style-type: none"> lengths and heights mass/weight capacity and volume time (hours, minutes, seconds) 	<ul style="list-style-type: none"> recognise and know the value of different denominations of coins and notes 	<ul style="list-style-type: none"> sequence events in chronological order using language [for example, before and after, next, first, today, yesterday, tomorrow, morning, afternoon and evening] recognise and use language relating to dates, including days of the week, weeks, months and years tell the time to the hour and half past the hour and draw the hands on a clock face to show these times 	<ul style="list-style-type: none"> describe position, direction and movement, including whole, half, quarter and three-quarter turns 	<ul style="list-style-type: none"> recognise and name common 2-D shapes [for example, rectangles (including squares), circles and triangles]
Autumn 2 Spring 2		Spring 4 Spring 5 Summer 6	Summer 5	Summer 6	Summer 3	Autumn 3

Addition and Subtraction

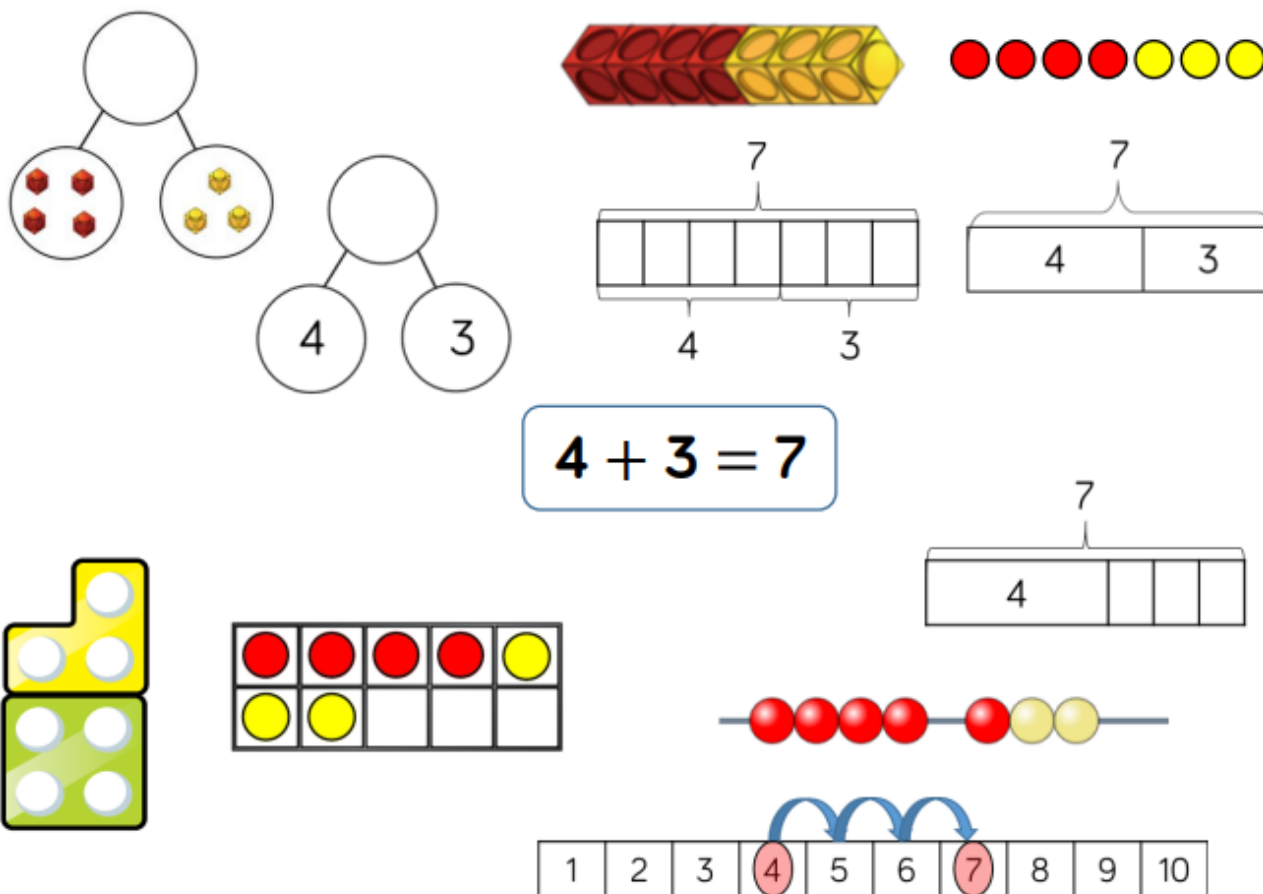
Add two 1-digit numbers to 10	1	Part-whole model Bar model Number shapes	Ten frames (within 10) Bead strings (10) Number tracks
Add 1 and 2-digit numbers to 20	1	Part-whole model Bar model Number shapes Ten frames (within 20)	Bead strings (20) Number tracks Number lines (labelled) Straws

Multiplication

Solve one-step problems with multiplication	1/2	Bar model Number shapes Counters	Ten frames Bead strings Number lines
Solve one-step problems with division (sharing)	1/2	Bar model Real life objects	Arrays Counters

Skill: Add 1-digit numbers within 10

Year: 1



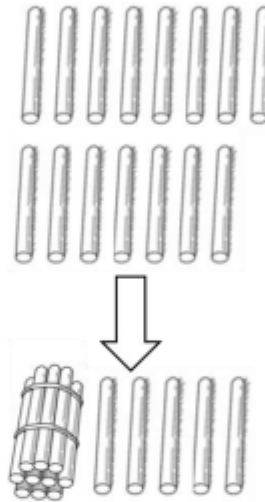
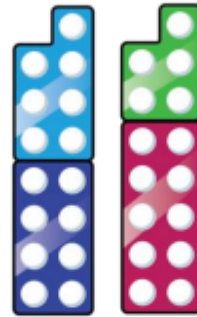
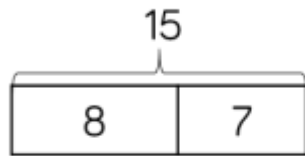
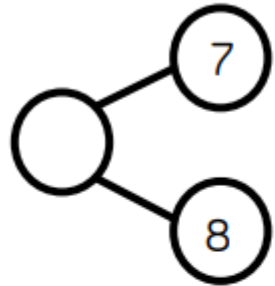
When adding numbers to 10, children can explore both aggregation and augmentation.

The part-whole model, discrete and continuous bar model, number shapes and ten frame support aggregation.

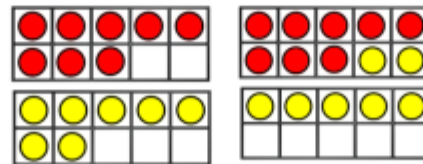
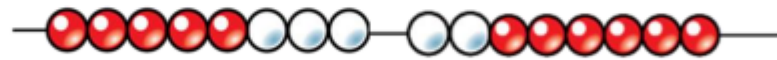
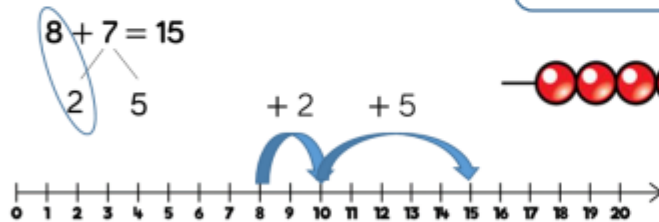
The combination bar model, ten frame, bead string and number track all support augmentation.

Skill: Add 1 and 2-digit numbers to 20

Year: 1/2



$$8 + 7 = 15$$

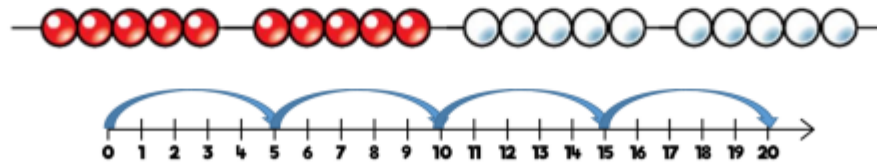
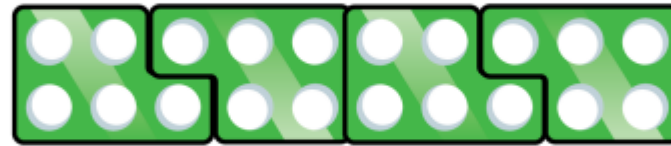
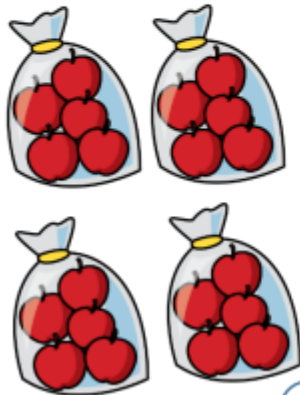


$$8 + 7 = 15$$

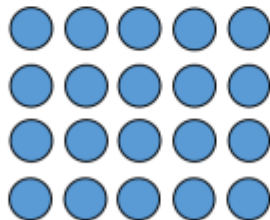
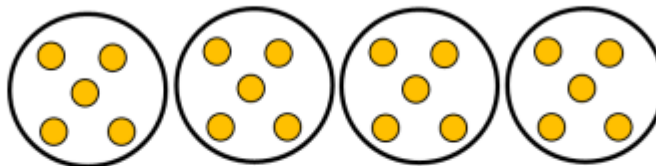
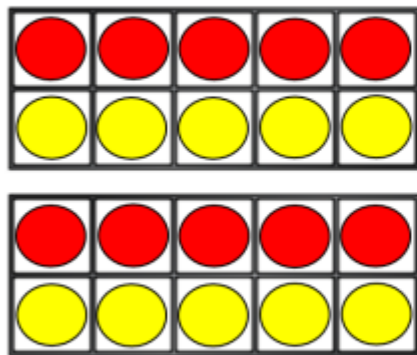
When adding one-digit numbers that cross 10, it is important to highlight the importance of ten ones equalling one ten. In Year 1, this is only done just by counting on. From Year 2, use different manipulatives can be used to represent this exchange alongside number lines to support children in understanding how to partition their jumps.

Skill: Solve 1-step problems using multiplication

Year: 1/2



One bag holds 5 apples.
How many apples do 4 bags hold?



$$5 + 5 + 5 + 5 = 20$$

$$4 \times 5 = 20$$

$$5 \times 4 = 20$$

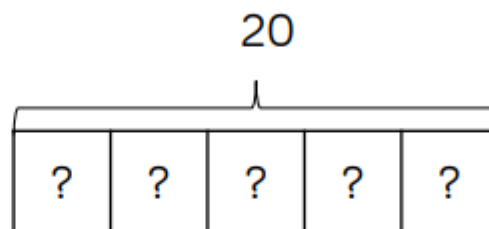
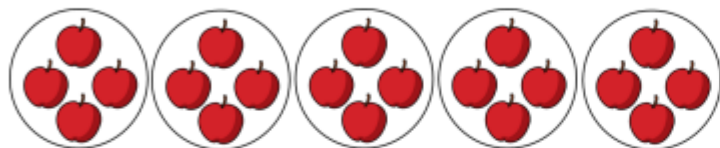
Children represent multiplication as repeated addition in many different ways.

In Year 1, children use concrete and pictorial representations to solve problems. They are not expected to record multiplication formally.

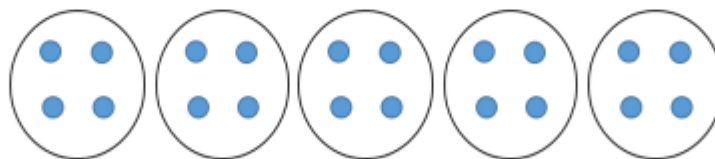
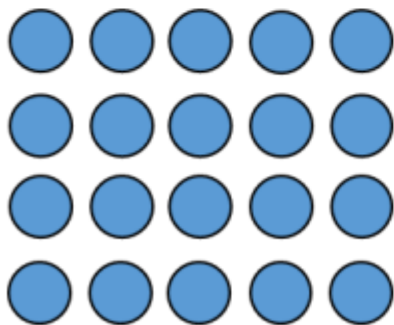
In Year 2, children are introduced to the multiplication symbol.

Skill: Solve 1-step problems using multiplication (sharing)

Year: 1/2



There are 20 apples altogether.
They are shared equally between 5 bags.
How many apples are in each bag?



$$20 \div 5 = 4$$

Children solve problems by sharing amounts into equal groups.

In Year 1, children use concrete and pictorial representations to solve problems. They are not expected to record division formally.

In Year 2, children are introduced to the division symbol.

Ready to progress criteria		Ready to progress criteria		<p style="text-align: center;">Ready to progress Year 1 to Year 2 objectives</p>	
<p>1NPV-1 Count within 100, forwards and backwards, starting with any number.</p>	<p>1AS-1 Compose numbers to 10 from 2 parts, and partition numbers to 10 into parts, including recognising odd and even numbers.</p>	Ready to progress criteria	<p>1NF-1 Develop fluency in addition and subtraction facts within 10</p>		
<p>1NPV-2 Reason about the location of numbers to 20 within the linear number system, including comparing using $<$ $>$ and $=$</p>	<p>1AS-2 Read, write and interpret equations containing addition (+), subtraction (-) and equals (=) symbols, and relate additive expressions and equations to real-life contexts.</p>		<p>1NF-2 Count forwards and backwards in multiples of 2, 5 and 10, up to 10 multiples, beginning with any multiple, and count forwards and backwards through the odd numbers.</p>		
Ready to progress criteria					
<p>1NF-1 Develop fluency in addition and subtraction facts within 10</p>					
<p>1NF-2 Count forwards and backwards in multiples of 2, 5 and 10, up to 10 multiples, beginning with any multiple, and count forwards and backwards through the odd numbers.</p>					

Addition and Subtraction

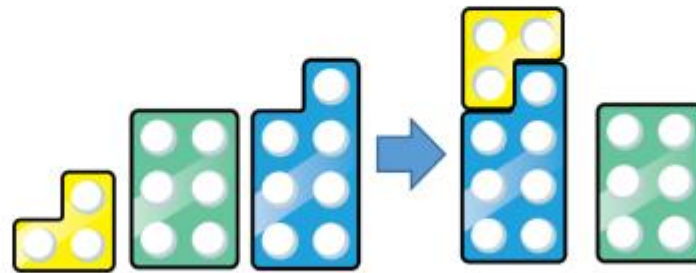
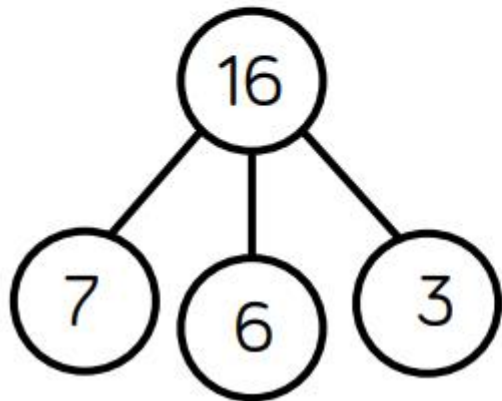
Add three 1-digit numbers	2	Part-whole model Bar model	Ten frames (within 20) Number shapes
Add 1 and 2-digit numbers to 100	2	Part-whole model Bar model Number lines (labelled)	Number lines (blank) Straws Hundred square
Add two 2-digit numbers	2	Part-whole model Bar model Number lines (blank) Straws	Base 10 Place value counters

Multiplication

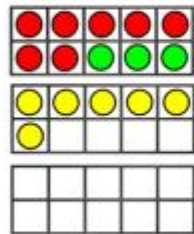
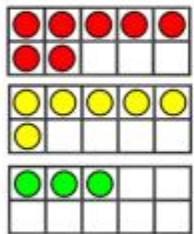
Solve one-step problems with multiplication	1/2	Bar model Number shapes Counters	Ten frames Bead strings Number lines
Solve one-step problems with division (sharing)	1/2	Bar model Real life objects	Arrays Counters

Skill: Add three 1-digit numbers

Year: 2

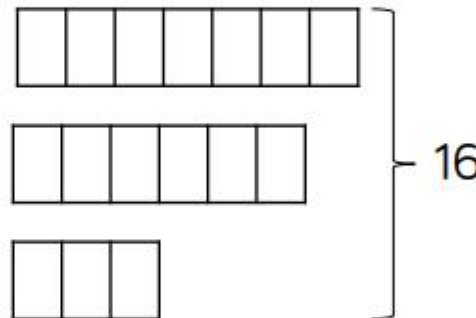


$$7 + 6 + 3 = 16$$



$$7 + 6 + 3 = 16$$

10



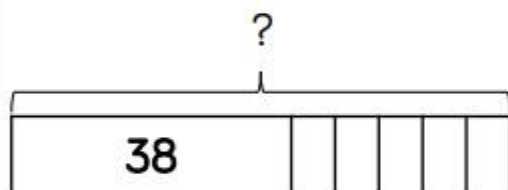
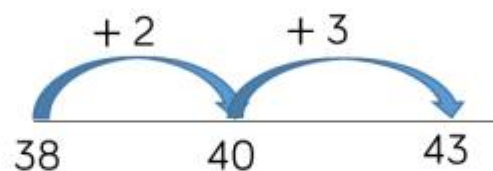
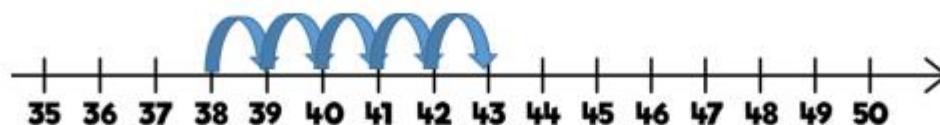
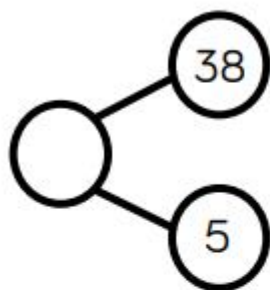
When adding three 1-digit numbers, children should be encouraged to look for number bonds to 10 or doubles to add the numbers more efficiently.

This supports children in their understanding of commutativity.

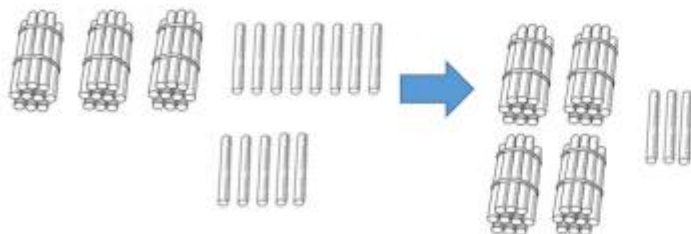
Manipulatives that highlight number bonds to 10 are effective when adding three 1-digit numbers.

Skill: Add 1-digit and 2-digit numbers to 100

Year: 2/3



$$38 + 5 = 43$$



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

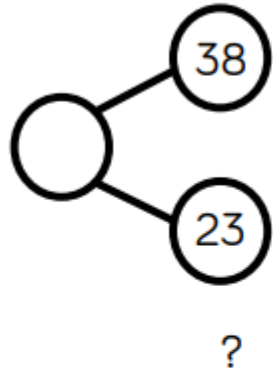
When adding single digits to a two-digit number, children should be encouraged to count on from the larger number.

They should also apply their knowledge of number bonds to add more efficiently e.g. $8 + 5 = 13$ so $38 + 5 = 43$.

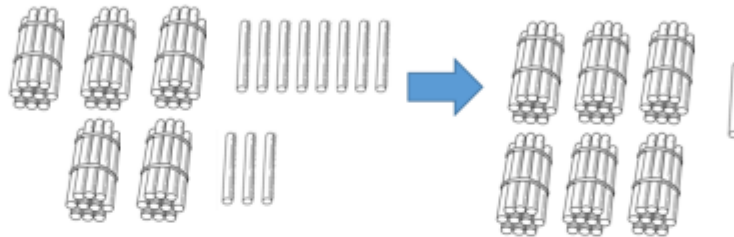
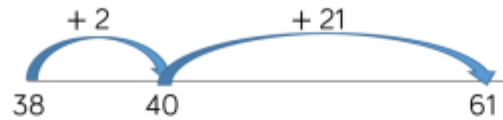
Hundred squares and straws can support children to find the number bond to 10.

Skill: Add two 2-digit numbers to 100

Year: 2/3



?	
38	23



$$38 + 23 = 61$$

Tens	Ones
	●●●● ●●●●
	●●●●

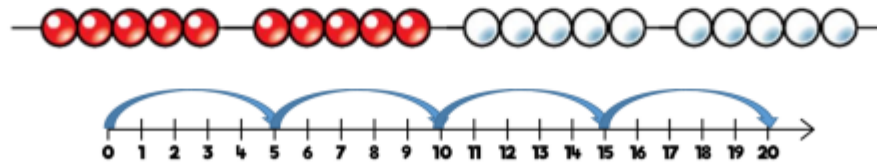
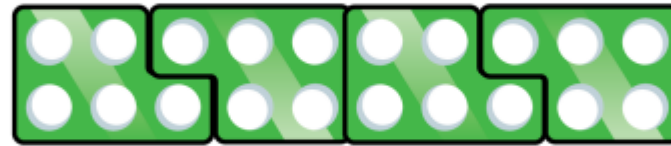
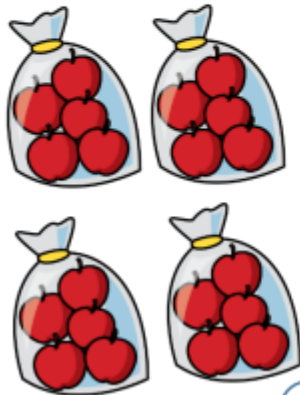
$$\begin{array}{r} 38 \\ + 23 \\ \hline 61 \\ 1 \end{array}$$

Tens	Ones
●10 ●10 ●10	●1 ●1 ●1 ●1 ●1 ●1 ●1 ●1
●10 ●10	●1 ●1 ●1
●10	

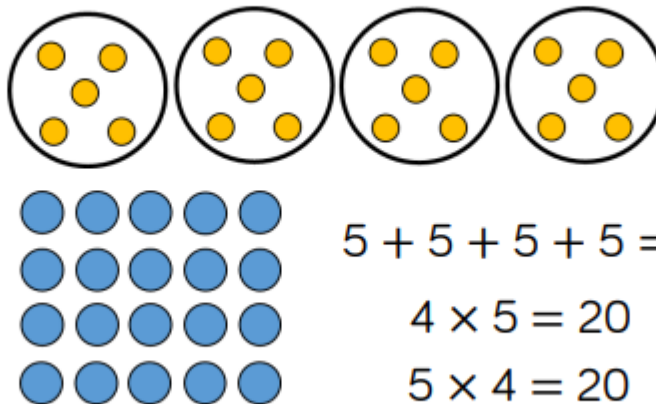
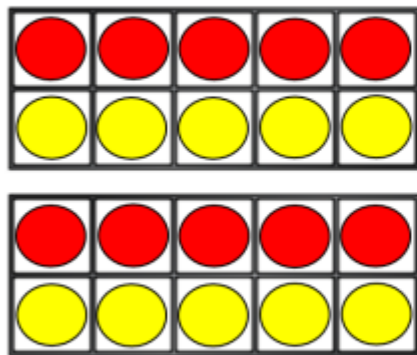
Children can use a blank number line and other representations to count on to find the total. Encourage them to jump to multiples of 10 to become more efficient. From Year 3, encourage children to use the formal column method when calculating alongside straws, base 10 or place value counters. As numbers become larger, straws become less efficient.

Skill: Solve 1-step problems using multiplication

Year: 1/2



One bag holds 5 apples.
How many apples do 4 bags hold?



$$5 + 5 + 5 + 5 = 20$$

$$4 \times 5 = 20$$

$$5 \times 4 = 20$$

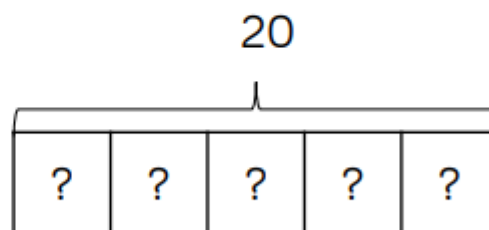
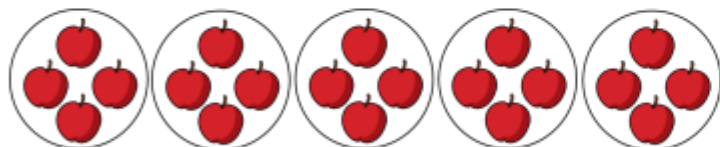
Children represent multiplication as repeated addition in many different ways.

In Year 1, children use concrete and pictorial representations to solve problems. They are not expected to record multiplication formally.

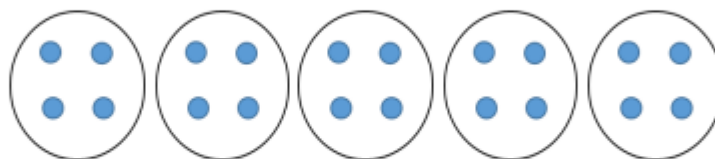
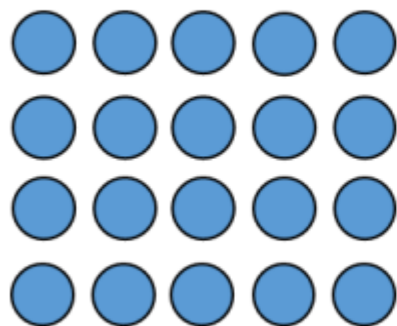
In Year 2, children are introduced to the multiplication symbol.

Skill: Solve 1-step problems using multiplication (sharing)

Year: 1/2



There are 20 apples altogether.
They are shared equally between 5 bags.
How many apples are in each bag?



$$20 \div 5 = 4$$

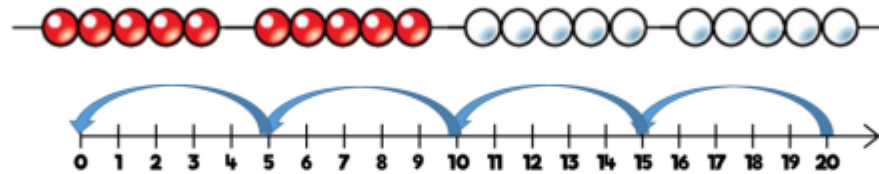
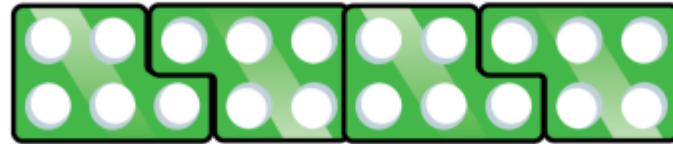
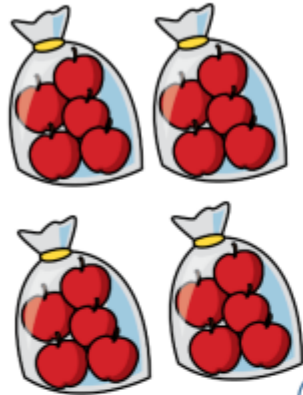
Children solve problems by sharing amounts into equal groups.

In Year 1, children use concrete and pictorial representations to solve problems. They are not expected to record division formally.

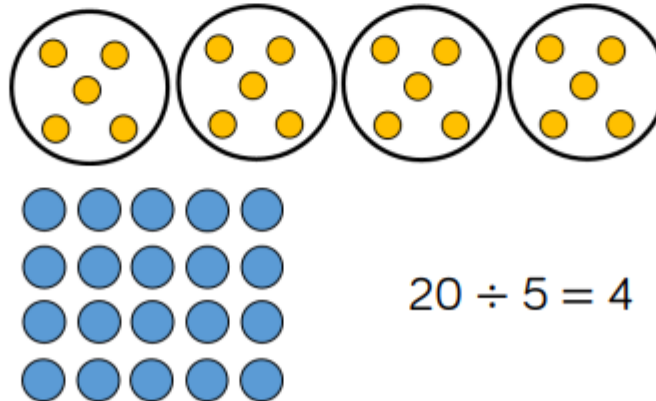
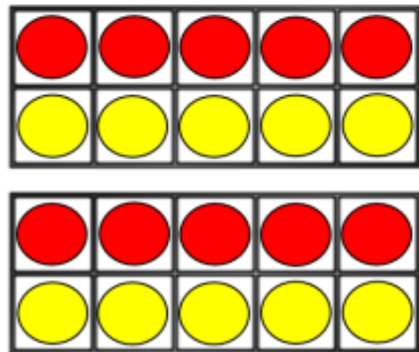
In Year 2, children are introduced to the division symbol.

Skill: Solve 1-step problems using division (grouping)

Year: 1/2



There are 20 apples altogether.
They are put in bags of 5.
How many bags are there?

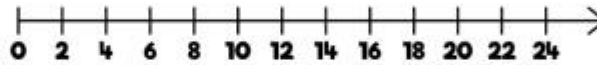
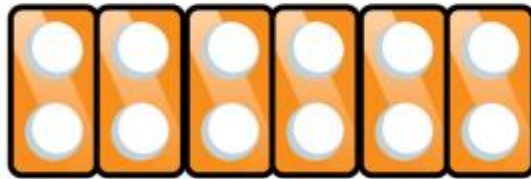


$$20 \div 5 = 4$$

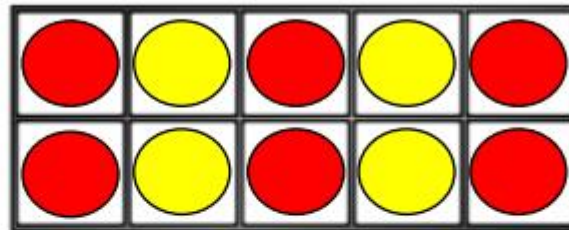
Children solve problems by grouping and counting the number of groups. Grouping encourages children to count in multiples and links to repeated subtraction on a number line. They can use concrete representations in fixed groups such as number shapes which helps to show the link between multiplication and division.

Skill: 2 times table

Year: 2



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50



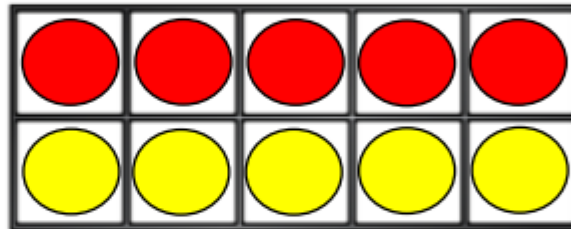
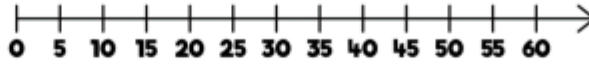
Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.

Look for patterns in the two times table, using concrete manipulatives to support. Notice how all the numbers are even and there is a pattern in the ones.

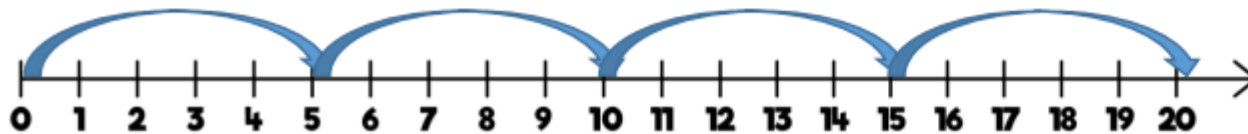
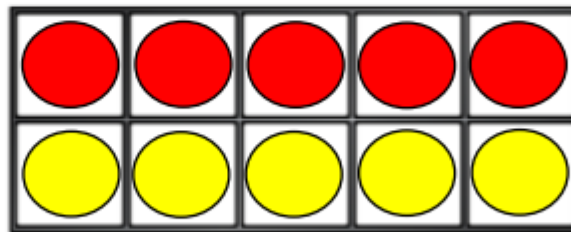
Use different models to develop fluency.

Skill: 5 times table

Year: 2



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

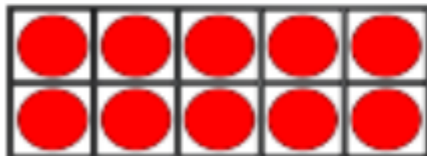
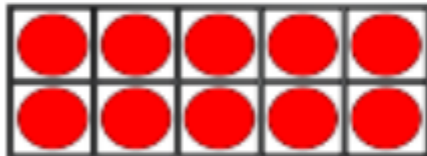
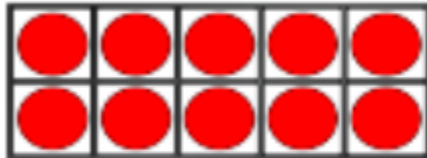
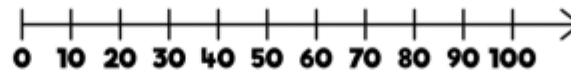


Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.

Look for patterns in the five times table, using concrete manipulatives to support. Notice the pattern in the ones as well as highlighting the odd, even, odd, even pattern.

Skill: 10 times table

Year: 2



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.

Look for patterns in the ten times table, using concrete manipulatives to support. Notice the pattern in the digits—the ones are always 0, and the tens increase by 1 ten each time.

Ready to progress
Year 2 to Year 3 objectives

Ready to progress criteria

2NF-1 Secure fluency in addition and subtraction facts within 10, through continued practice.

Ready to progress criteria

2NPV-1 Recognise the place value of each digit in two-digit numbers, and compose and decompose two-digit numbers using standard and non-standard partitioning.

2NPV-2 Reason about the location of any two-digit number in the linear number system, including identifying the previous and next multiple of 10

Ready to progress criteria

2AS-1 Add and subtract across 10

2AS-2 Recognise the subtraction structure of 'difference' and answer questions of the form, "How many more...?".

2AS-3 Add and subtract within 100 by applying related one-digit addition and subtraction facts: add and subtract only ones or only tens to/from a two-digit number.

2AS-4 Add and subtract within 100 by applying related one-digit addition and subtraction facts: add and subtract any 2 two-digit numbers.

Ready to progress criteria

2MD-1 Recognise repeated addition contexts, representing them with multiplication equations and calculating the product, within the 2, 5 and 10 multiplication tables.

2MD-2 Relate grouping problems where the number of groups is unknown to multiplication equations with a missing factor, and to division equations (quotitive division).

Addition and Subtraction

Add with up to 3-digits	3	Part-whole model Bar model	Base 10 Place value counters Column addition
-------------------------	---	-------------------------------	--

Subtract with up to 3-digits	3	Part-whole model Bar model	Base 10 Place value counters Column subtraction
------------------------------	---	-------------------------------	---

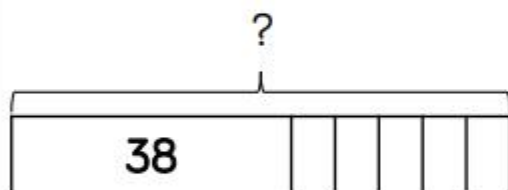
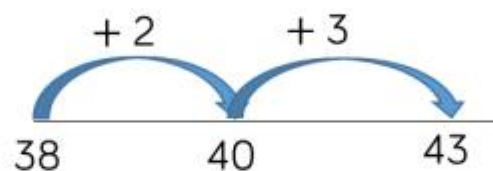
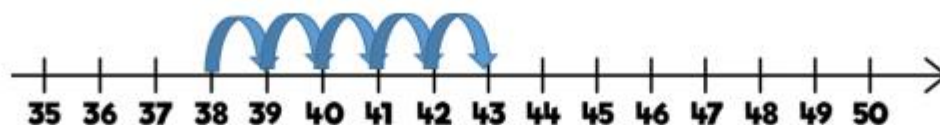
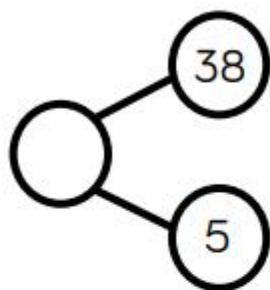
Multiplication

Multiply 2-digit by 1-digit numbers	3/4	Place value counters Base 10	Expanded written method Short written method
-------------------------------------	-----	---------------------------------	---

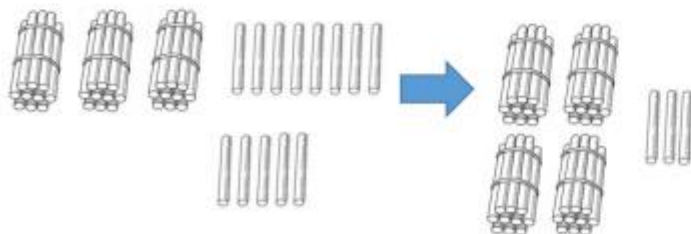
Divide 2-digits by 1-digit (no exchange sharing)	3	Straws Base 10 Bar model	Place value counters Part-whole model
Divide 2-digits by 1-digit (sharing with exchange)	3	Straws Base 10 Bar model	Place value counters Part-whole model
Divide 2-digits by 1-digit (sharing with remainders)	3/4	Straws Base 10 Bar model	Place value counters Part-whole model

Skill: Add 1-digit and 2-digit numbers to 100

Year: 2/3



$$38 + 5 = 43$$



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

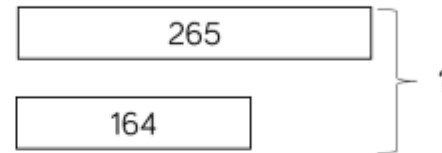
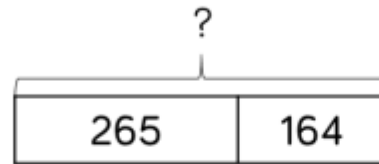
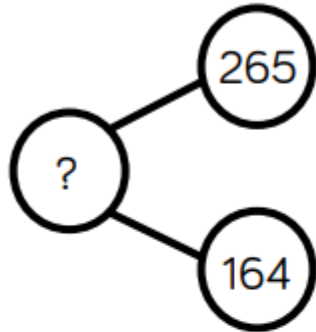
When adding single digits to a two-digit number, children should be encouraged to count on from the larger number.

They should also apply their knowledge of number bonds to add more efficiently e.g. $8 + 5 = 13$ so $38 + 5 = 43$.

Hundred squares and straws can support children to find the number bond to 10.

Skill: Add numbers with up to 3 digits

Year: 3



$$265 + 164 = 429$$

Hundreds	Tens	Ones

$$\begin{array}{r} 265 \\ + 164 \\ \hline 429 \\ 1 \end{array}$$

Hundreds	Tens	Ones

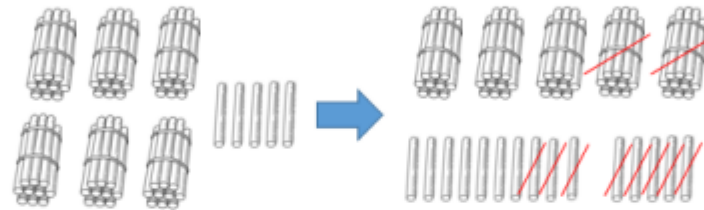
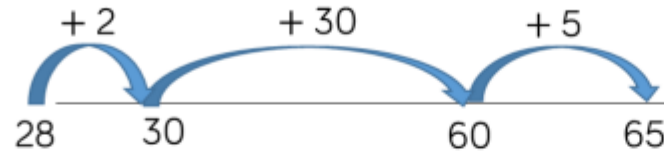
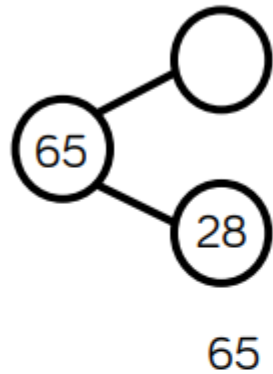
Base 10 and place value counters are the most effective manipulatives when adding numbers with up to 3 digits.

Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

Plain counters on a place value grid can also be used to support learning.

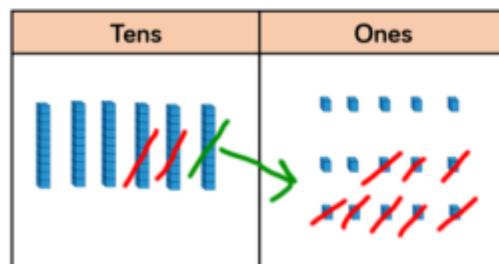
Skill: Subtract 1 and 2-digit numbers to 100

Year: 2/3

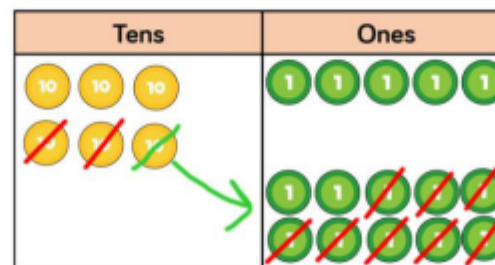


65	
?	28

$$65 - 28 = 37$$



$$\begin{array}{r} 5 \ 1 \\ 65 \\ - 28 \\ \hline 37 \end{array}$$



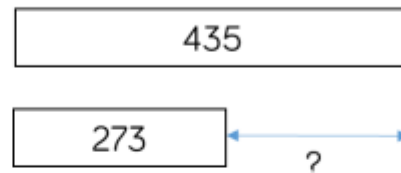
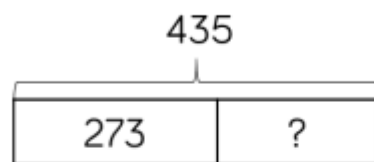
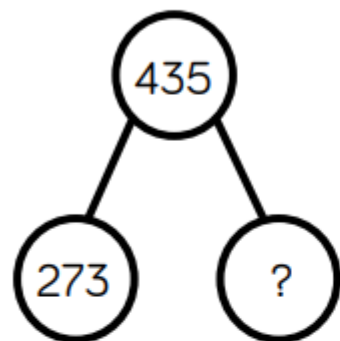
Children can also use a blank number line to count back to find the difference.

Encourage them to jump to multiples of 10 to become more efficient.

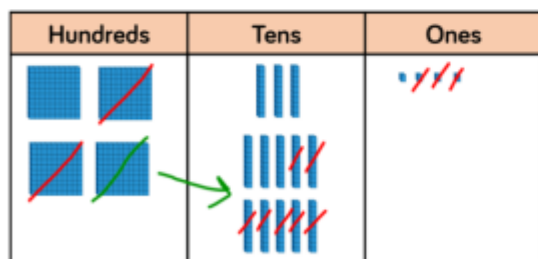
From Year 3, encourage children to use the formal column method when calculating alongside straws, base 10 or place value counters. As numbers become larger, straws become less efficient.

Skill: Subtract numbers with up to 3 digits

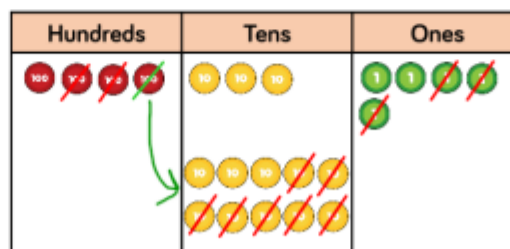
Year: 3



$$435 - 273 = 162$$



$$\begin{array}{r} ^3 ^1 \\ 435 \\ - 273 \\ \hline 162 \end{array}$$



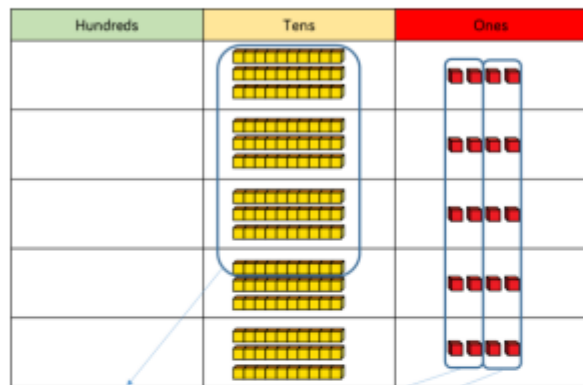
Base 10 and place value counters are the most effective manipulative when subtracting numbers with up to 3 digits.

Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

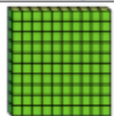
Plain counters on a place value grid can also be used to support learning.

Skill: Multiply 2-digit numbers by 1-digit numbers

Year: 3/4

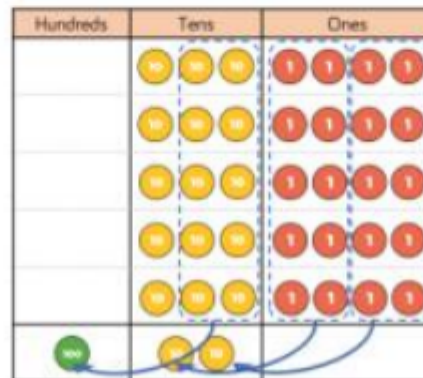


	H	T	O	
		3	4	
x			5	
		2	0	(5 × 4)
+	1	5	0	(5 × 30)
	1	7	0	



$$34 \times 5 = 170$$

	H	T	O
		3	4
x			5
	1	7	0
	1	2	



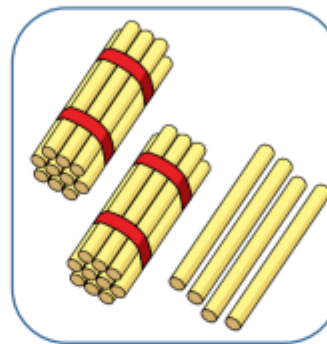
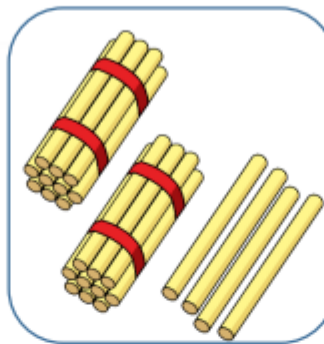
Informal methods and the expanded method are used in Year 3 before moving on to the short multiplication method in Year 4.

Place value counters should be used to support the understanding of the method rather than supporting the multiplication, as children should use times table knowledge.

Skill: Divide 2-digits by 1-digit (sharing with no exchange)

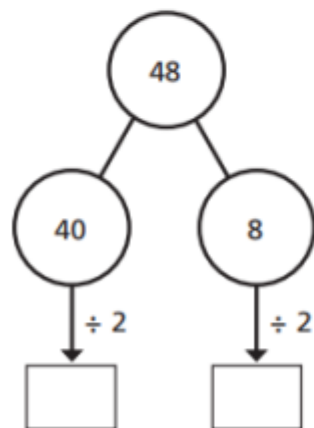
Year: 3

Tens	Ones
10 10	1 1 1 1
10 10	1 1 1 1

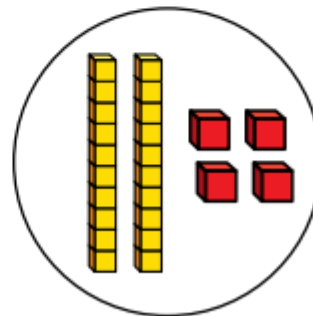
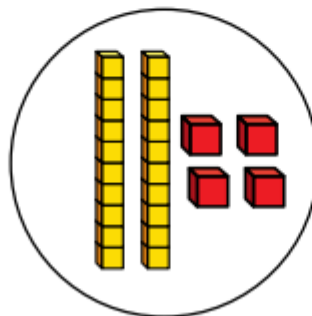


When dividing larger numbers, children can use manipulatives that allow them to partition into tens and ones.

Straws, Base 10 and place value counters can all be used to share numbers into equal groups.



$$48 \div 2 = 24$$











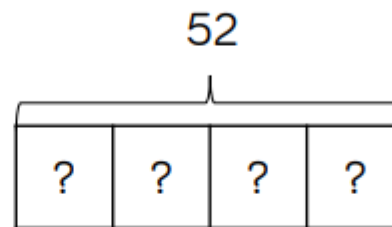
Part-whole models can provide children with a clear written method that matches the concrete representation.

Skill: Divide 2-digits by 1-digit (sharing with exchange)

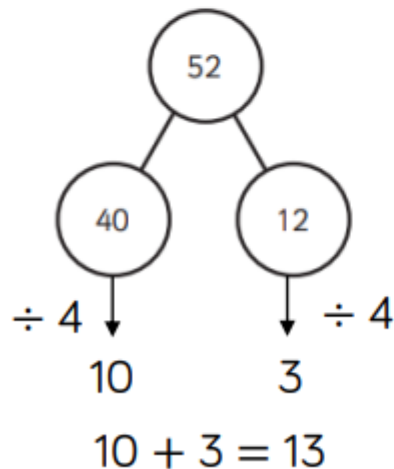

Year: 3/4








Tens	Ones
	
	
	
	



$$52 \div 4 = 13$$

Tens	Ones
	
	
	
	

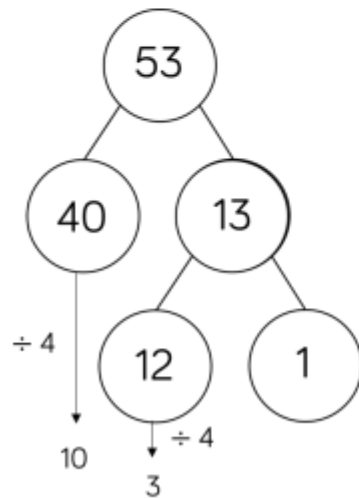
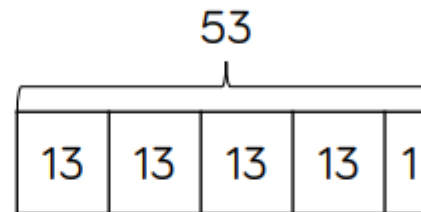
When dividing numbers involving an exchange, children can use Base 10 and place value counters to exchange one ten for ten ones.

Children should start with the equipment outside the place value grid before sharing the tens and ones equally between the rows.

Flexible partitioning in a part-whole model supports this method.

Skill: Divide 2-digits by 1-digit (sharing with remainders)

Year: 3/4



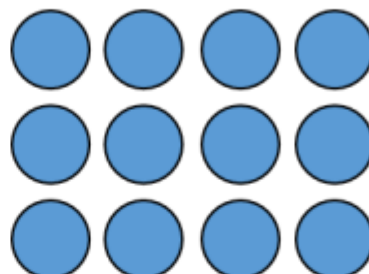
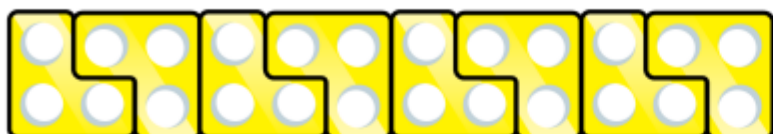
$$53 \div 4 = 13 \text{ r}1$$



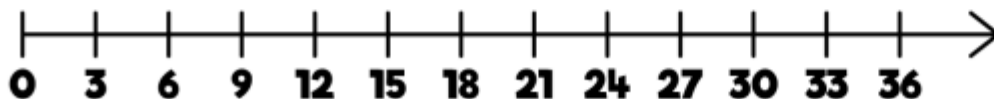
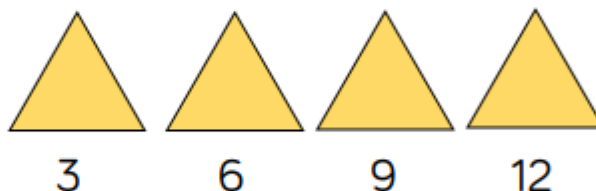
When dividing numbers with remainders, children can use Base 10 and place value counters to exchange one ten for ten ones. Starting with the equipment outside the place value grid will highlight remainders, as they will be left outside the grid once the equal groups have been made. Flexible partitioning in a part-whole model supports this method.

Skill: 3 times table

Year: 3



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

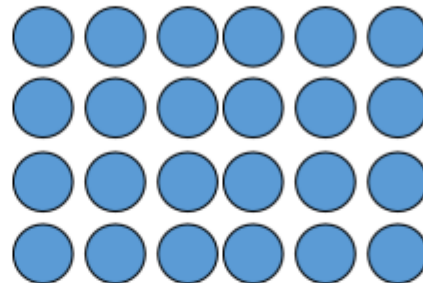


Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.

Look for patterns in the three times table, using concrete manipulatives to support. Notice the odd, even, odd, even pattern using number shapes to support. Highlight the pattern in the ones using a hundred square.

Skill: 4 times table

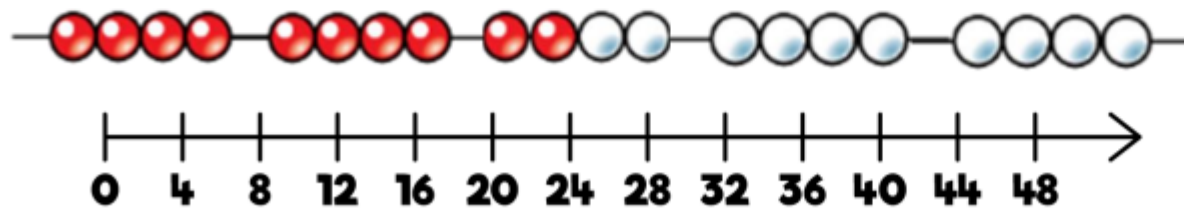
Year: 3



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50



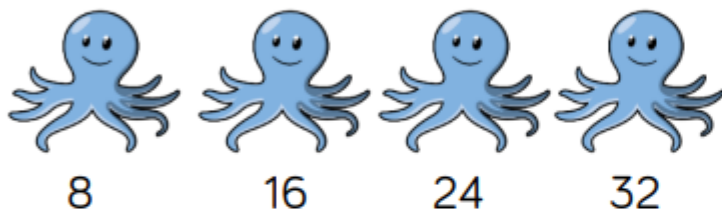
4	8	12	16	20
24	28	32	36	40
44	48	52	56	60



Encourage daily counting in multiples, supported by a number line or a hundred square. Look for patterns in the four times table, using manipulatives to support. Make links to the 2 times table, seeing how each multiple is double the twos. Notice the pattern in the ones within each group of five multiples. Highlight that all the multiples are even using number shapes to support.

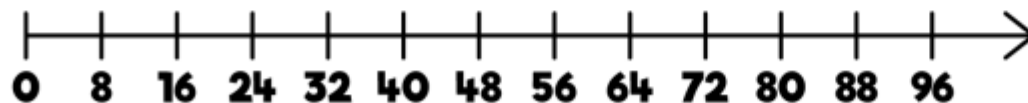
Skill: 8 times table

Year: 3



8	16	24	32	40
48	56	64	72	80

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



Encourage daily counting in multiples, supported by a number line or a hundred square. Look for patterns in the eight times table, using manipulatives to support. Make links to the 4 times table, seeing how each multiple is double the fours. Notice the pattern in the ones within each group of five multiples. Highlight that all the multiples are even using number shapes to support.

Ready to progress
Year 3 to Year 4 objectives

Ready to progress criteria		Ready to progress criteria	
3AS-1 Calculate complements to 100			3NPV-1 Know that 10 tens are equivalent to 1 hundred, and that 100 is 10 times the size of 10; apply this to identify and work out how many 10s there are in other three-digit multiples of 10
3AS-2 Add and subtract up to three-digit numbers using columnar methods.		Ready to progress criteria	3NPV-2 Recognise the place value of each digit in three-digit numbers, and compose and decompose three-digit numbers using standard and non-standard partitioning.
		3NF-1 Secure fluency in addition and subtraction facts that bridge 10, through continued practice.	3NPV-3 Reason about the location of any three-digit number in the linear number system, including identifying the previous and next multiple of 100 and 10
3AS-3 Manipulate the additive relationship: Understand the inverse relationship between addition and subtraction, and how both relate to the part-part-whole structure. Understand and use the commutative property of addition, and understand the related property for subtraction.		3NF-2 Recall multiplication facts, and corresponding division facts, in the 10, 5, 2, 4 and 8 multiplication tables, and recognise products in these multiplication tables as multiples of the corresponding number.	3NPV-4 Divide 100 into 2, 4, 5 and 10 equal parts, and read scales/number lines marked in multiples of 100 with 2, 4, 5 and 10 equal parts.
		3NF-3 Apply place-value knowledge to known additive and multiplicative number facts (scaling facts by 10).	

Addition and Subtraction

Add with up to 4-digits	4	Part-whole model Bar model	Base 10 Place value counters Column addition
-------------------------	---	-------------------------------	--

Subtract with up to 4-digits	4	Part-whole model Bar model	Base 10 Place value counters Column subtraction
------------------------------	---	-------------------------------	---

Multiplication and division

Multiply 2-digit by 1-digit numbers	3/4	Place value counters Base 10	Expanded written method Short written method
Multiply 3-digit by 1-digit numbers	4	Place value counters Base 10	Short written method

Divide 2-digits by 1-digit (sharing with remainders)	3/4	Straws Base 10 Bar model	Place value counters Part-whole model
--	-----	--------------------------------	--

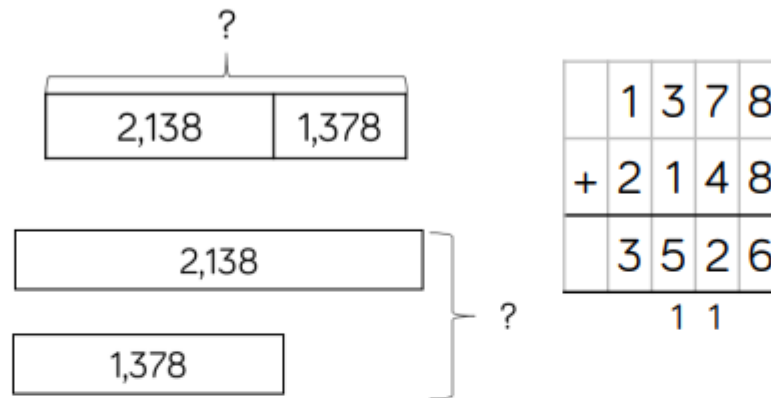
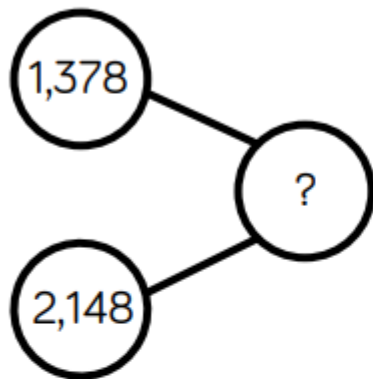
Divide 2-digits by 1-digit (grouping)	4/5	Place value counters Counters	Place value grid Written short division
Divide 3-digits by 1-digit (sharing with exchange)	4	Base 10 Bar model	Place value counters Part-whole model
Divide 3-digits by 1-digit (grouping)	4/5	Place value counters Counters	Place value grid Written short division

Recall and use multiplication and division facts for the 6-times table	4	Hundred square Number shapes	Bead strings Number tracks Everyday objects
--	---	---------------------------------	---

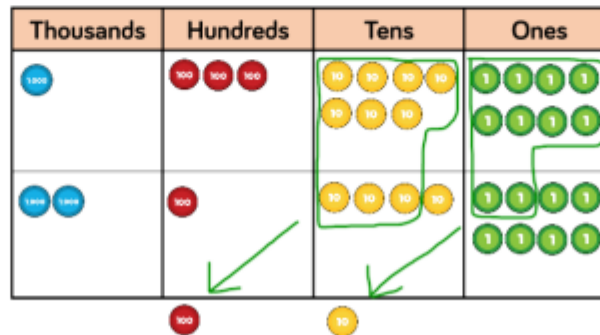
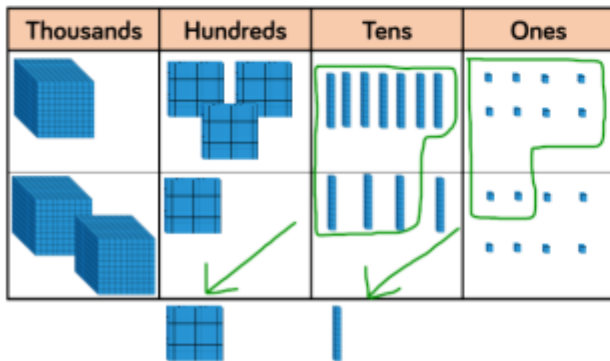
Skill	Year	Representations and models	
Recall and use multiplication and division facts for the 7-times table	4	Hundred square Number shapes	Bead strings Number lines
Recall and use multiplication and division facts for the 9-times table	4	Hundred square Number shapes	Bead strings Number lines
Recall and use multiplication and division facts for the 11-times table	4	Hundred square Base 10	Place value counters Number lines
Recall and use multiplication and division facts for the 12-times table	4	Hundred square Base 10	Place value counters Number lines

Skill: Add numbers with up to 4 digits

Year: 4



$$1,378 + 2,148 = 3,526$$



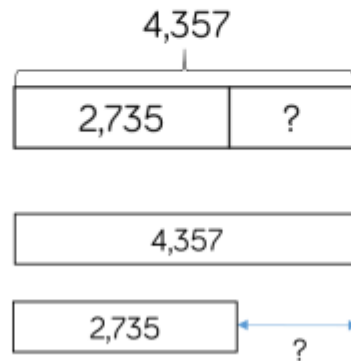
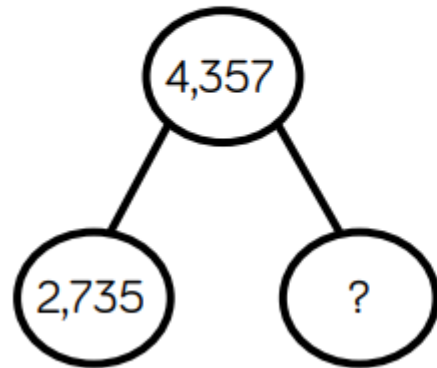
Base 10 and place value counters are the most effective manipulatives when adding numbers with up to 4 digits.

Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

Plain counters on a place value grid can also be used to support learning.

Skill: Subtract numbers with up to 4 digits

Year: 4



$$\begin{array}{r} \overset{3}{4} \overset{1}{3} 57 \\ - 2735 \\ \hline 1622 \end{array}$$

$$4,357 - 2,735 = 1,622$$

Base 10 and place value counters are the most effective manipulatives when subtracting numbers with up to 4 digits.

Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

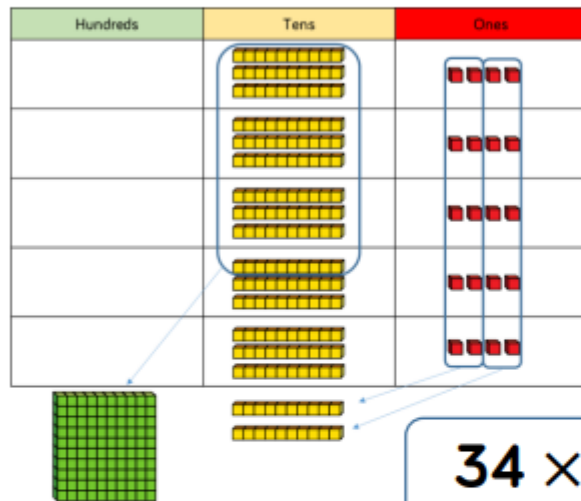
Plain counters on a place value grid can also be used to support learning.

Thousands	Hundreds	Tens	Ones

Thousands	Hundreds	Tens	Ones

Skill: Multiply 2-digit numbers by 1-digit numbers

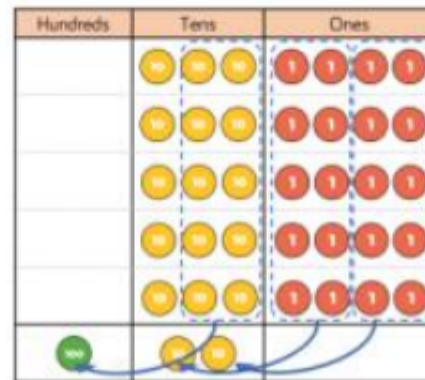
Year: 3/4



	H	T	O	
		3	4	
x			5	
		2	0	(5 × 4)
+	1	5	0	(5 × 30)
	1	7	0	

$$34 \times 5 = 170$$

	H	T	O
		3	4
x			5
	1	7	0
	1	2	

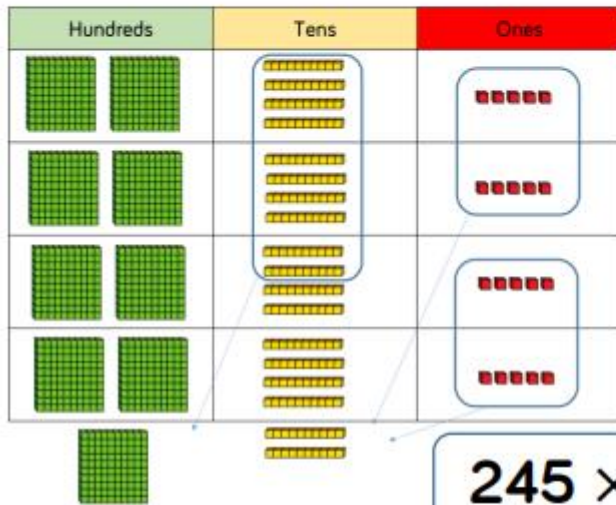


Informal methods and the expanded method are used in Year 3 before moving on to the short multiplication method in Year 4.

Place value counters should be used to support the understanding of the method rather than supporting the multiplication, as children should use times table knowledge.

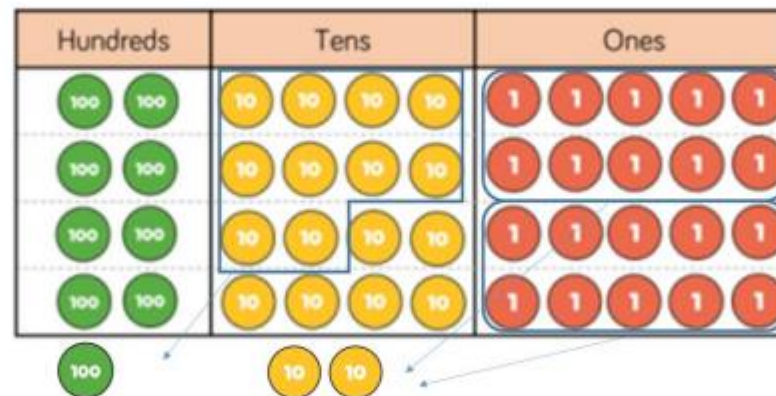
Skill: Multiply 3-digit numbers by 1-digit numbers

Year: 4



	H	T	O
	2	4	5
x			4
	9	8	0
	1	2	

$$245 \times 4 = 980$$











When moving to 3-digit by 1-digit multiplication, encourage children to move towards the short, formal written method.

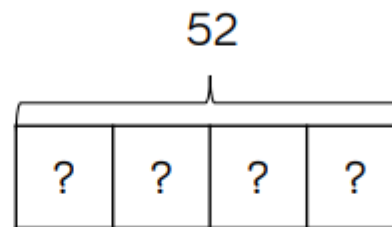
Base 10 and place value counters continue to support the understanding of the written method. Limit the number of exchanges needed in the questions and move children away from resources when multiplying larger numbers.

Skill: Divide 2-digits by 1-digit (sharing with exchange)

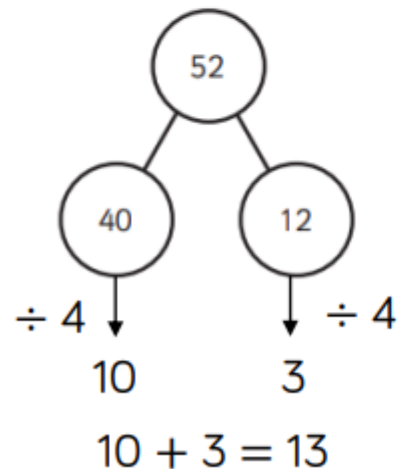
Year: 3/4














Tens	Ones
	
	
	
	



$$52 \div 4 = 13$$



Tens	Ones
	  
	  
	  
	  

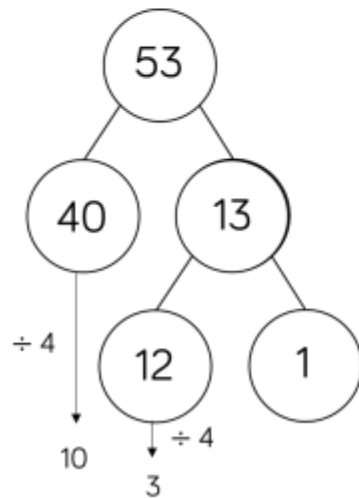
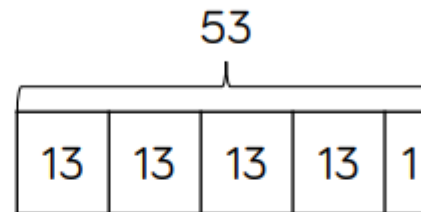
When dividing numbers involving an exchange, children can use Base 10 and place value counters to exchange one ten for ten ones.

Children should start with the equipment outside the place value grid before sharing the tens and ones equally between the rows.

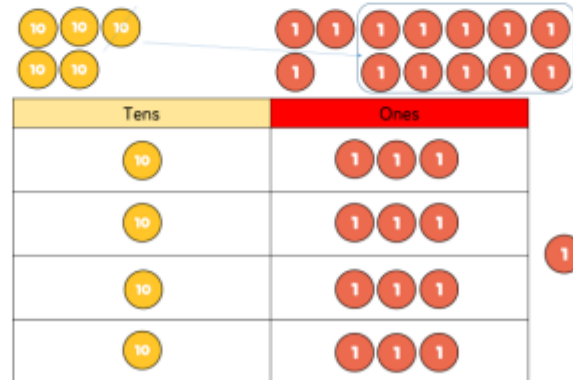
Flexible partitioning in a part-whole model supports this method.

Skill: Divide 2-digits by 1-digit (sharing with remainders)

Year: 3/4



$$53 \div 4 = 13 \text{ r}1$$

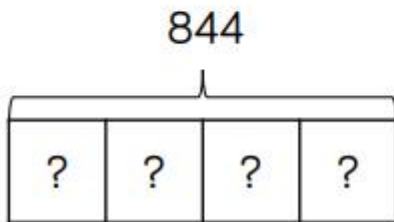


When dividing numbers with remainders, children can use Base 10 and place value counters to exchange one ten for ten ones. Starting with the equipment outside the place value grid will highlight remainders, as they will be left outside the grid once the equal groups have been made. Flexible partitioning in a part-whole model supports this method.

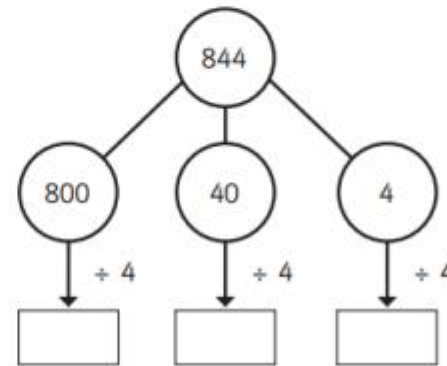
Skill: Divide 3-digits by 1-digit (sharing)

Year: 4

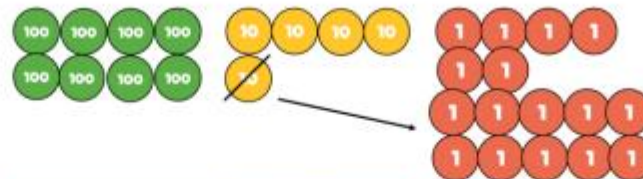
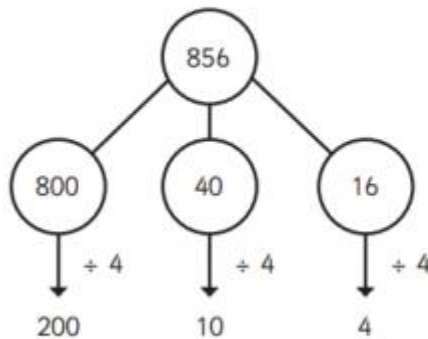
$$844 \div 4 = 211$$



H	T	O
100 100	10	1
100 100	10	1
100 100	10	1
100 100	10	1



$$856 \div 4 = 214$$



Hundreds	Tens	Ones
100 100	10	1 1 1 1
100 100	10	1 1 1 1
100 100	10	1 1 1 1
100 100	10	1 1 1 1

Children can continue to use place value counters to share 3-digit numbers into equal groups. Children should start with the equipment outside the place value grid before sharing the hundreds, tens and ones equally between the rows. This method can also help to highlight remainders. Flexible partitioning in a part-whole model supports this method.

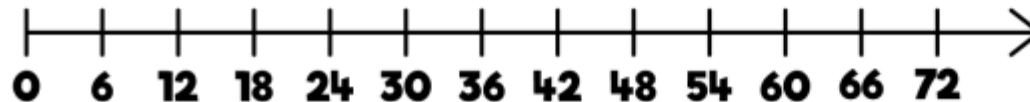
Skill: 6 times table

Year: 4



6	12	18	24	30
36	42	48	54	60
66	72	78	84	90

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



Encourage daily counting in multiples, supported by a number line or a hundred square. Look for patterns in the six times table, using manipulatives to support. Make links to the 3 times table, seeing how each multiple is double the threes. Notice the pattern in the ones within each group of five multiples. Highlight that all the multiples are even using number shapes to support.

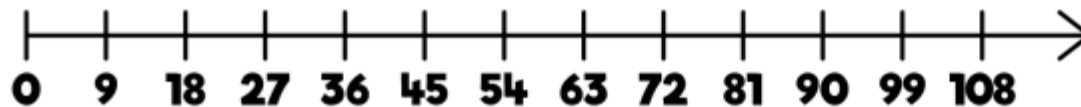
Skill: 9 times table

Year: 4



9	18	27	36	45
54	63	72	81	90

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square. Look for patterns in the nine times table, using concrete manipulatives to support. Notice the pattern in the tens and ones using the hundred square to support as well as noting the odd, even pattern within the multiples.

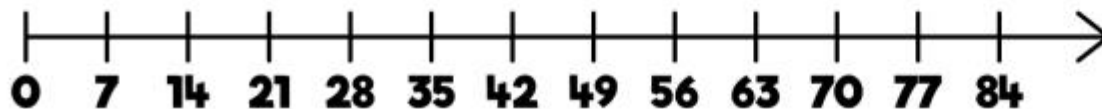
Skill: 7 times table

Year: 4



7	14	21	28	35
42	49	56	63	70

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



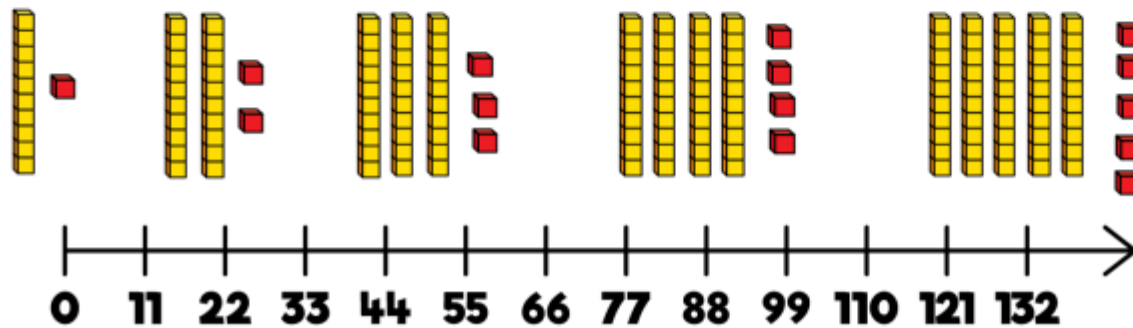
Encourage daily counting in multiples both forwards and backwards, supported by a number line or a hundred square. The seven times table can be trickier to learn due to the lack of obvious pattern in the numbers, however they already know several facts due to commutativity. Children can still see the odd, even pattern in the multiples using number shapes to support.

Skill: 11 times table

Year: 4

11	22	33	44	55	66
77	88	99	110	121	132

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.

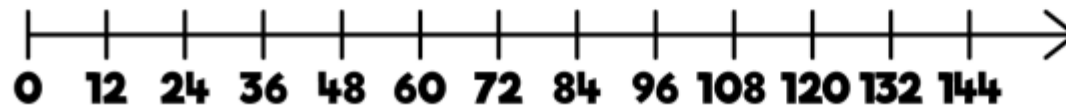
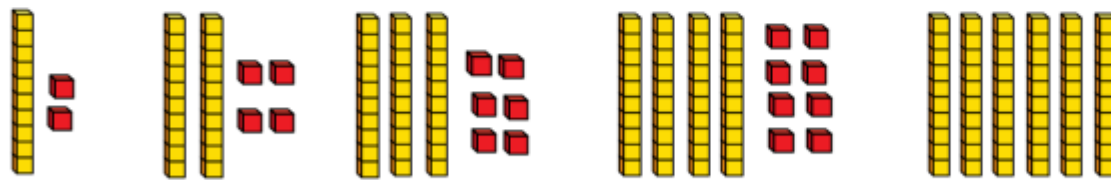
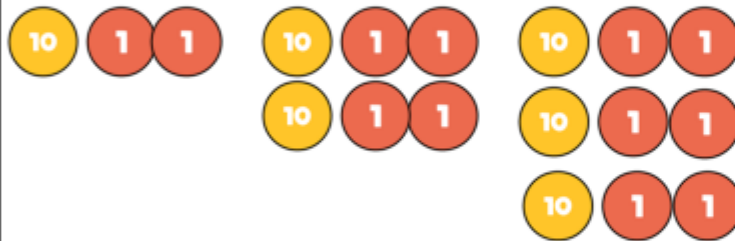
Look for patterns in the eleven times table, using concrete manipulatives to support. Notice the pattern in the tens and ones using the hundred square to support. Also consider the pattern after crossing 100

Skill: 12 times table

Year: 4

12	24	36	48	60
72	84	96	108	120
132	144			

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



Encourage daily counting in multiples, supported by a number line or a hundred square. Look for patterns in the 12 times table, using manipulatives to support. Make links to the 6 times table, seeing how each multiple is double the sixes. Notice the pattern in the ones within each group of five multiples. The hundred square can support in highlighting this pattern.

Ready to progress
Year 4 to Year 5 objectives

Ready to progress criteria
4NF-1 Recall multiplication and division facts up to 12×12 and recognise products in multiplication tables as multiples of the corresponding number.
4NF-2 Solve division problems, with two-digit dividends and one-digit divisors, that involve remainders, and interpret remainders appropriately according to the context.
4NF-3 Apply place-value knowledge to known additive and multiplicative number facts (scaling facts by 100).

Ready to progress criteria
4NPV-1 Know that 10 hundreds are equivalent to 1 thousand, and that 1,000 is 10 times the size of 100; apply this to identify and work out how many 100s there are in other four-digit multiples of 100
4NPV-2 Recognise the place value of each digit in four-digit numbers, and compose and decompose four-digit numbers using standard and non-standard partitioning.
4NPV-3 Reason about the location of any four-digit number in the linear number system, including identifying the previous and next multiple of 1,000 and 100, and rounding to the nearest of each.
4NPV-4 Divide 1,000 into 2, 4, 5 and 10 equal parts, and read scales/number lines marked in multiples of 1,000 with 2, 4, 5 and 10 equal parts.

Ready to progress criteria
4MD-1 Multiply and divide whole numbers by 10 and 100 (keeping to whole number quotients); understand this as equivalent to making a number 10 or 100 times the size.
4MD-2 Manipulate multiplication and division equations, and understand and apply the commutative property of multiplication.
4MD-3 Understand and apply the distributive property of multiplication.

Add with more than 4 digits	5	Part-whole model Bar model	Place value counters Column addition
Add with up to 3 decimal places	5	Part-whole model Bar model	Place value counters Column addition
Subtract with more than 4 digits	5	Part-whole model Bar model	Place value counters Column subtraction
Subtract with up to 3 decimal places	5	Part-whole model Bar model	Place value counters Column subtraction

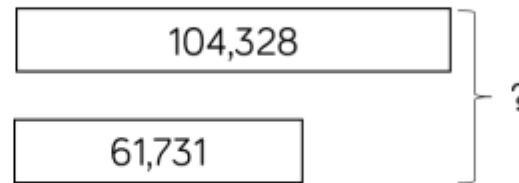
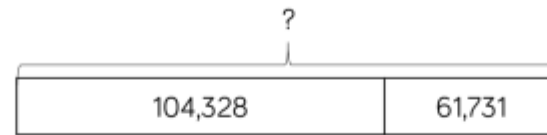
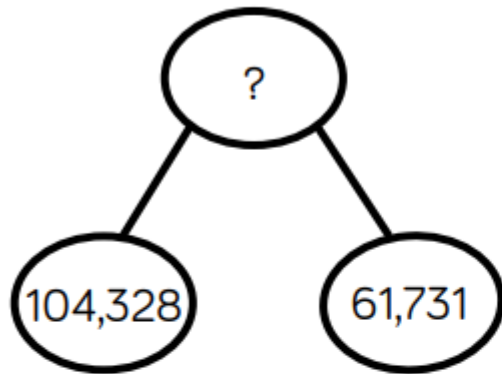
Multiplication and division

Multiply 4-digit by 1-digit numbers	5	Place value counters	Short written method
Multiply 2-digit by 2-digit numbers	5	Place value counters Base 10	Short written method Grid method
Multiply 2-digit by 3-digit numbers	5	Place value counters	Short written method Grid method
Multiply 2-digit by 4-digit numbers	5/6	Formal written method	

Divide 2-digits by 1-digit (grouping)	4/5	Place value counters Counters	Place value grid Written short division
Divide 3-digits by 1-digit (grouping)	4/5	Place value counters Counters	Place value grid Written short division
Divide 4-digits by 1-digit (grouping)	5	Place value counters Counters	Place value grid Written short division

Skill: Add numbers with more than 4 digits

Year: 5/6



$$104,328 + 61,731 = 166,059$$

HTh	TTh	Th	H	T	O
100000		1000 1000 1000 1000	100 100 100	10 10	1 1 1 1 1 1 1 1
	10000 10000 10000 10000 10000 10000	1000	100 100 100 100 100 100 100	10 10 10	1

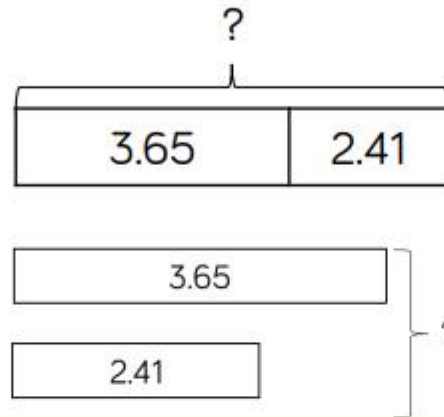
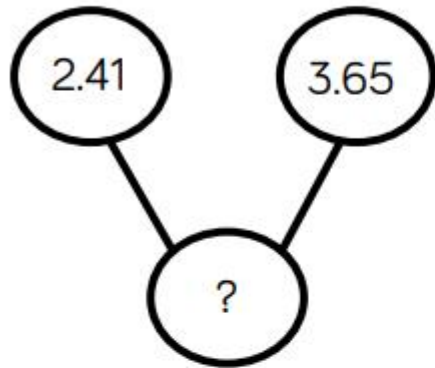
1	0	4	3	2	8
+	6	1	7	3	1
1	6	6	0	5	9
					1

Place value counters or plain counters on a place value grid are the most effective concrete resources when adding numbers with more than 4 digits.

At this stage, children should be encouraged to work in the abstract, using the column method to add larger numbers efficiently.

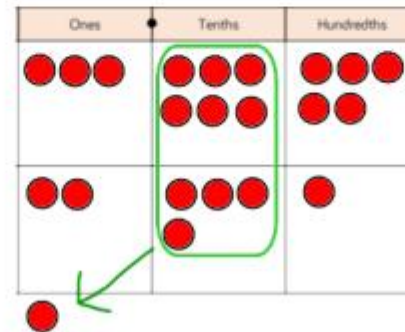
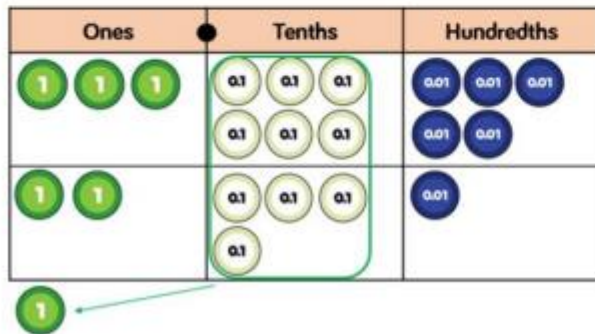
Skill: Add with up to 3 decimal places

Year: 5



$$\begin{array}{r} 3.65 \\ + 2.41 \\ \hline 6.06 \\ 1 \end{array}$$

$$3.65 + 2.41 = 6.06$$

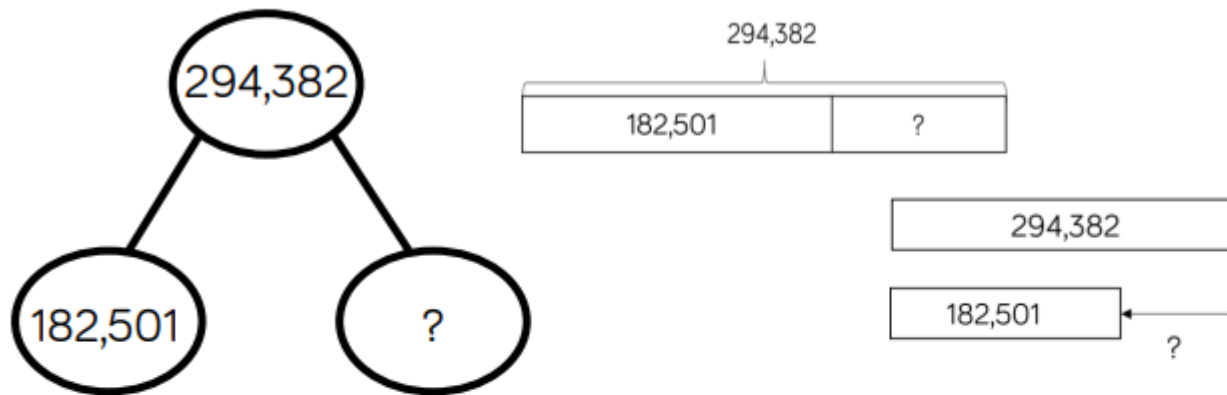


Place value counters and plain counters on a place value grid are the most effective manipulatives when adding decimals with 1, 2 and then 3 decimal places.

Ensure children have experience of adding decimals with a variety of decimal places. This includes putting this into context when adding money and other measures.

Skill: Subtract numbers with more than 4 digits

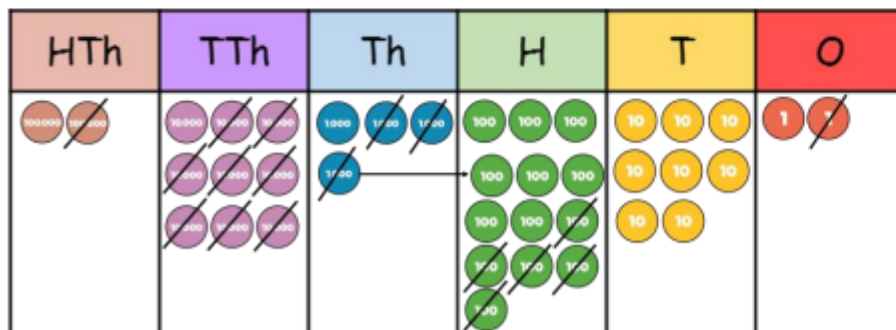
Year: 5/6



$$294,382 - 182,501 = 111,881$$

Place value counters or plain counters on a place value grid are the most effective concrete resource when subtracting numbers with more than 4 digits.

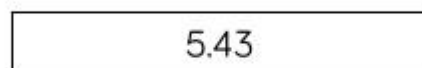
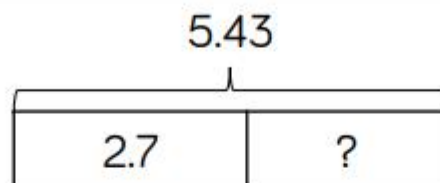
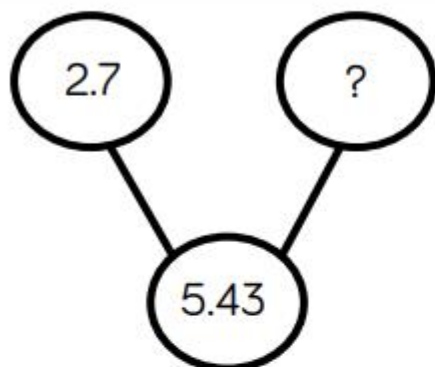
At this stage, children should be encouraged to work in the abstract, using column method to subtract larger numbers efficiently.



	2	9	3	13	8	2
-	1	8	2	5	0	1
	1	1	1	8	8	1

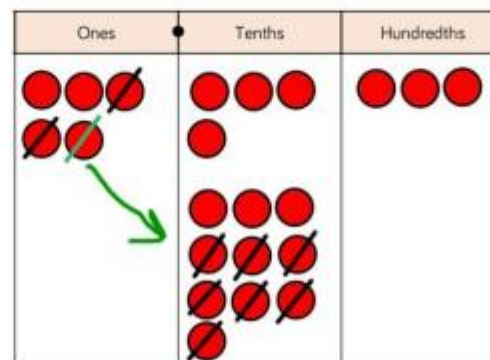
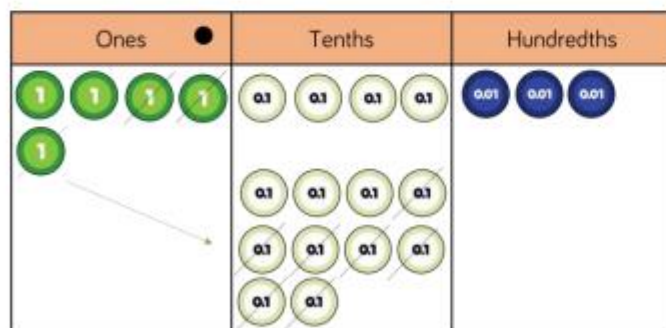
Skill: Subtract with up to 3 decimal places

Year: 5/6



$$\begin{array}{r} 4 \quad 1 \\ 5.43 \\ - 2.7 \\ \hline 2.73 \end{array}$$

$$5.43 - 2.7 = 2.73$$

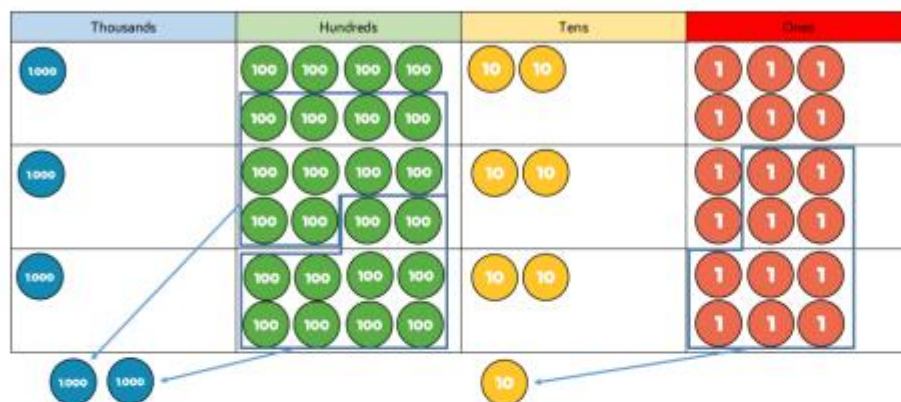


Place value counters and plain counters on a place value grid are the most effective manipulative when subtracting decimals with 1, 2 and then 3 decimal places.

Ensure children have experience of subtracting decimals with a variety of decimal places. This includes putting this into context when subtracting money and other measures.

Skill: Multiply 4-digit numbers by 1-digit numbers

Year: 5



$$1,826 \times 3 = 5,478$$

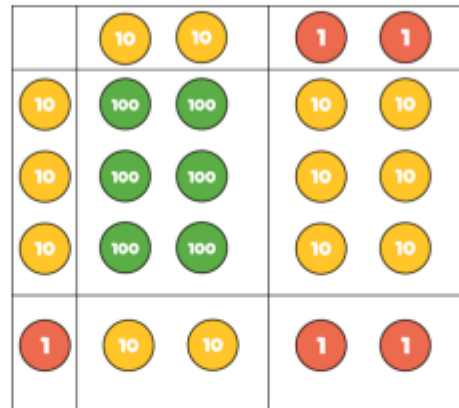
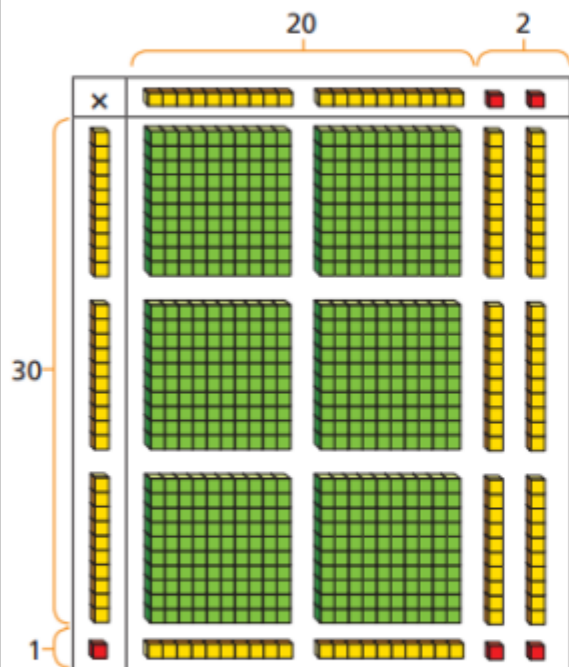
	Th	H	T	O
	1	8	2	6
×				3
	5	4	7	8

2 1

When multiplying 4-digit numbers, place value counters are the best manipulative to use to support children in their understanding of the formal written method. If children are multiplying larger numbers and struggling with their times tables, encourage the use of multiplication grids so children can focus on the use of the written method.

Skill: Multiply 2-digit numbers by 2-digit numbers

Year: 5



×	20	2
30	600	60
1	20	2

	H	T	O
		2	2
×		3	1
		2	2
	6	6	0
	6	8	2

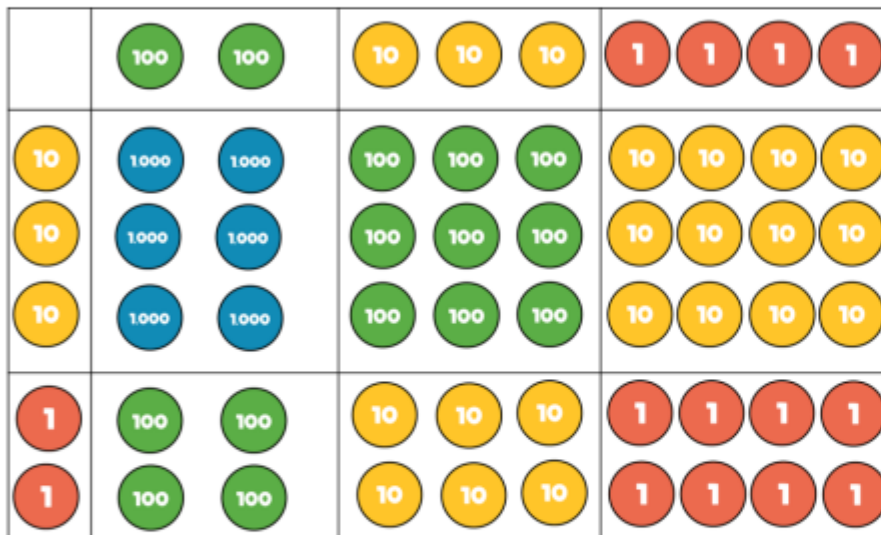
$$22 \times 31 = 682$$

When multiplying a multi-digit number by 2-digits, use the area model to help children understand the size of the numbers they are using. This links to finding the area of a rectangle by finding the space covered by the Base 10.

The grid method matches the area model as an initial written method before moving on to the formal written multiplication method.

Skill: Multiply 3-digit numbers by 2-digit numbers

Year: 5



	Th	H	T	O
		2	3	4
×			3	2
		4	6	8
1 7	1 0	2	0	
7	4	8	8	

×	200	30	4
30	6,000	900	120
2	400	60	8

$$234 \times 32 = 7,488$$

Children can continue to use the area model when multiplying 3-digits by 2-digits. Place value counters become more efficient to use but Base 10 can be used to highlight the size of numbers.

Children should now move towards the formal written method, seeing the links with the grid method.

Skill: Multiply 4-digit numbers by 2-digit numbers

Year: 5/6

	TTh	Th	H	T	O
		2	7	3	9
×				2	8
	2	1	9	1	2
	₂	₅	₃	₇	
	5	4	7	8	0
	₁		₁		
	7	6	6	9	2
			₁		

$$2,739 \times 28 = 76,692$$

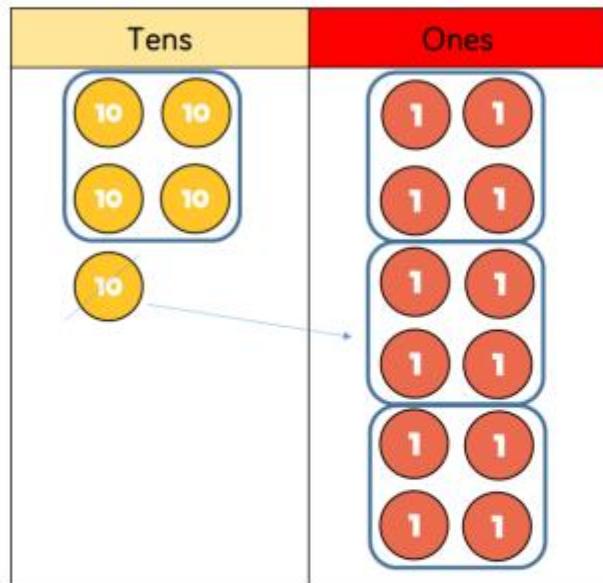
When multiplying 4-digits by 2-digits, children should be confident in using the formal written method.

If they are still struggling with times tables, provide multiplication grids to support when they are focusing on the use of the method.

Consider where exchanged digits are placed and make sure this is consistent.

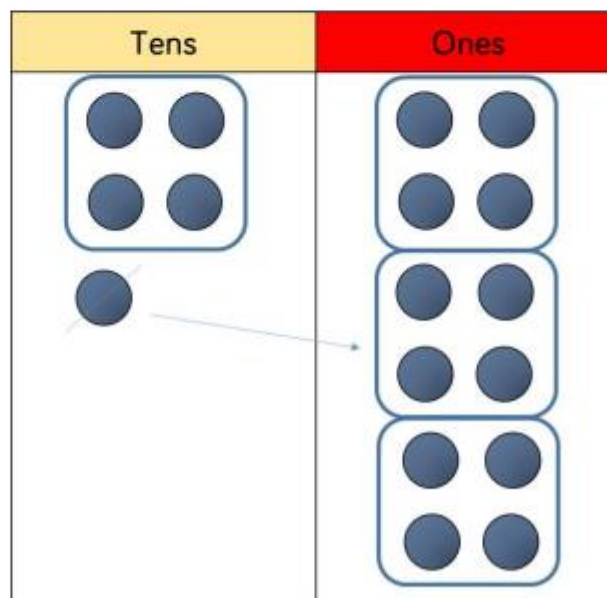
Skill: Divide 2-digits by 1-digit (grouping)

Year: 5



		1	3	
	4	5	12	

A vertical line is drawn between the 5 and 12 in the second row.



$$52 \div 4 = 13$$

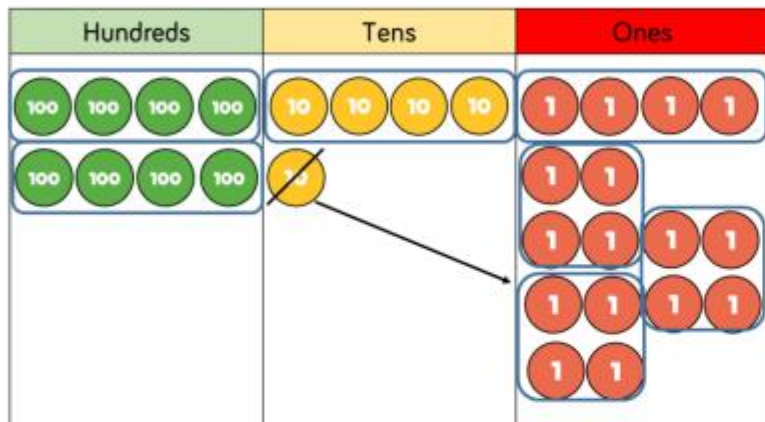
When using the short division method, children use grouping. Starting with the largest place value, they group by the divisor.

Language is important here. Children should consider 'How many groups of 4 tens can we make?' and 'How many groups of 4 ones can we make?'

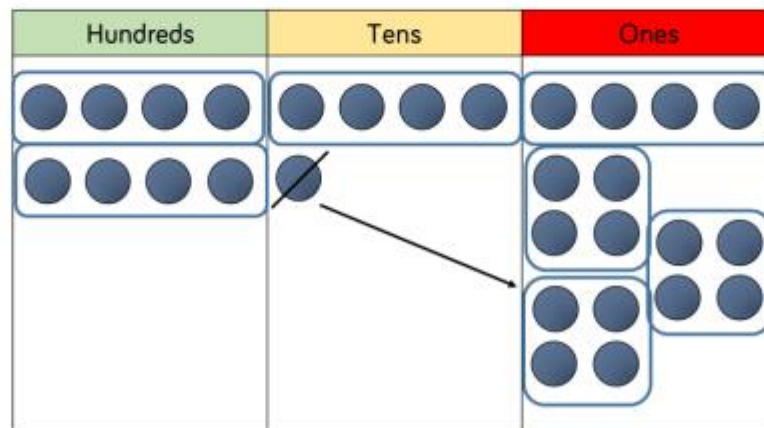
Remainders can also be seen as they are left ungrouped.

Skill: Divide 3-digits by 1-digit (grouping)

Year: 5



		2	1	4
	4	8	5	16



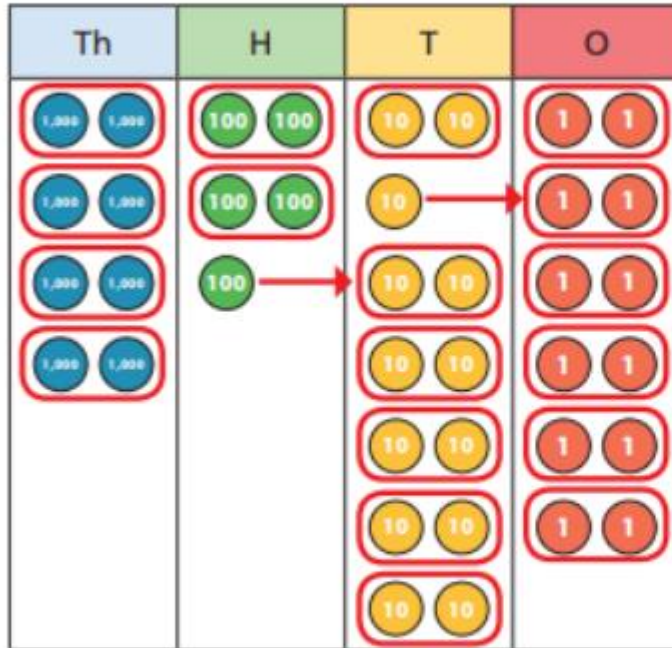
$$856 \div 4 = 214$$

Children can continue to use grouping to support their understanding of short division when dividing a 3-digit number by a 1-digit number.

Place value counters or plain counters can be used on a place value grid to support this understanding. Children can also draw their own counters and group them through a more pictorial method.

Skill: Divide 4-digits by 1-digit (grouping)

Year: 5



	4	2	6	6
2	8	5	13	12

$$8,532 \div 2 = 4,266$$

Place value counters or plain counters can be used on a place value grid to support children to divide 4-digits by 1-digit. Children can also draw their own counters and group them through a more pictorial method.

Children should be encouraged to move away from the concrete and pictorial when dividing numbers with multiple exchanges.

Ready to progress
Year 5 to Year 6 objectives

Ready to progress criteria
SNPV-1 Know that 10 tenths are equivalent to 1 one, and that 1 is 10 times the size of 0.1. Know that 100 hundredths are equivalent to 1 one, and that 1 is 100 times the size of 0.01. Know that 10 hundredths are equivalent to 1 tenth, and that 0.1 is 10 times the size of 0.01
SNPV-2 Recognise the place value of each digit in numbers with up to 2 decimal places, and compose and decompose numbers with up to 2 decimal places using standard and non-standard partitioning.
SNPV-3 Reason about the location of any number with up to 2 decimal places in the linear number system, including identifying the previous and next multiple of 1 and 0.1 and rounding to the nearest of each.
SNPV-4 Divide 1 into 2, 4, 5 and 10 equal parts, and read scales/number lines marked in units of 1 with 2, 4, 5 and 10 equal parts.
SNPV-5 Convert between units of measure, including using common decimals and fractions.

Ready to progress criteria
SNF-1 Secure fluency in multiplication table facts, and corresponding division facts, through continued practice.
SNF-2 Apply place-value knowledge to known additive and multiplicative number facts (scaling facts by 1 tenth or 1 hundredth).

Ready to progress criteria
5MD-1 Multiply and divide numbers by 10 and 100; understand this as equivalent to making a number 10 or 100 times the size, or 1 tenth or 1 hundredth times the size.
5MD-2 Find factors and multiples of positive whole numbers, including common factors and common multiples, and express a given number as a product of 2 or 3 factors.
5MD-3 Multiply any whole number with up to 4 digits by any one-digit number using a formal written method.
5MD-4 Divide a number with up to 4 digits by a one-digit number using a formal written method, and interpret remainders appropriately for the context.

Addition and Subtraction

No new objectives here

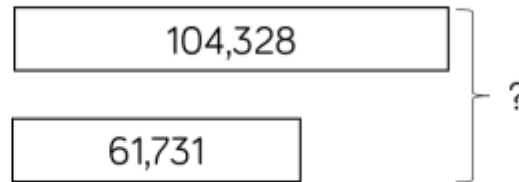
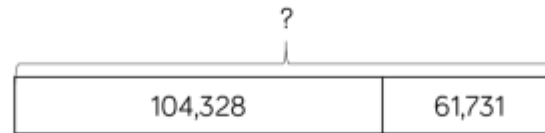
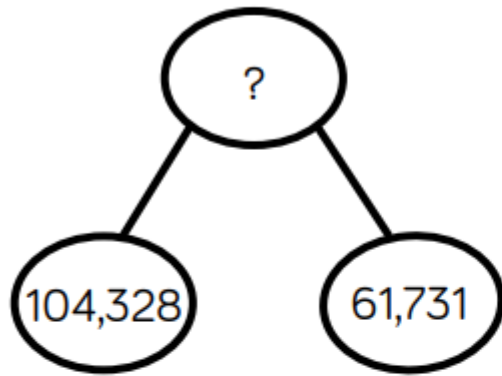
Multiplication and division

Multiply 2-digit by 4-digit numbers	5/6	Formal written method
-------------------------------------	-----	-----------------------

Divide multi-digits by 2-digits (short division)	6	Written short division	List of multiples
Divide multi-digits by 2-digits (long division)	6	Written long division	List of multiples

Skill: Add numbers with more than 4 digits

Year: 5/6



$$104,328 + 61,731 = 166,059$$

HTh	TTh	Th	H	T	O
100000		1000 1000 1000 1000	100 100 100	10 10	1 1 1 1 1 1 1 1
	10000 10000 10000 10000 10000 10000	1000	100 100 100 100 100 100 100	10 10 10	1

1	0	4	3	2	8
+	6	1	7	3	1
1	6	6	0	5	9

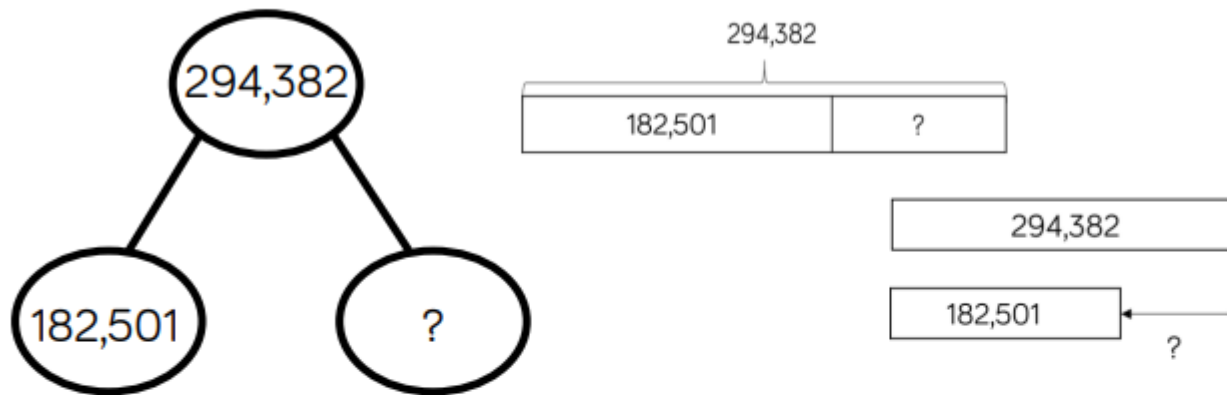
1

Place value counters or plain counters on a place value grid are the most effective concrete resources when adding numbers with more than 4 digits.

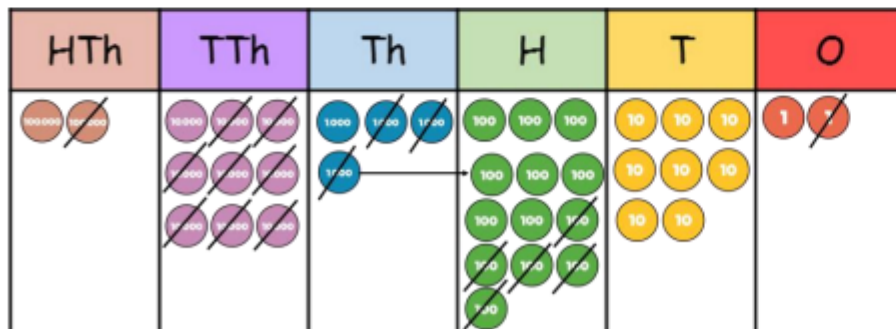
At this stage, children should be encouraged to work in the abstract, using the column method to add larger numbers efficiently.

Skill: Subtract numbers with more than 4 digits

Year: 5/6



$$294,382 - 182,501 = 111,881$$



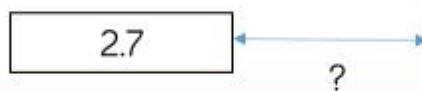
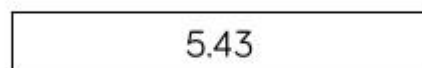
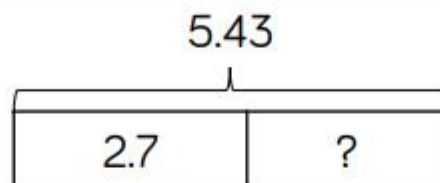
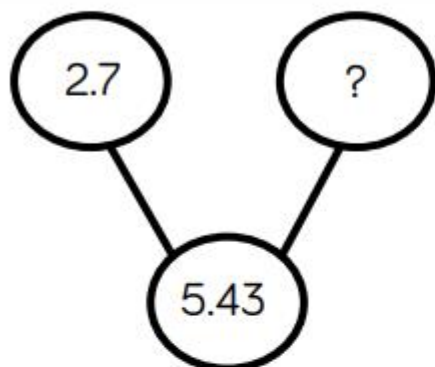
	2	9	3	13	8	2
-	1	8	2	5	0	1
	1	1	1	8	8	1

Place value counters or plain counters on a place value grid are the most effective concrete resource when subtracting numbers with more than 4 digits.

At this stage, children should be encouraged to work in the abstract, using column method to subtract larger numbers efficiently.

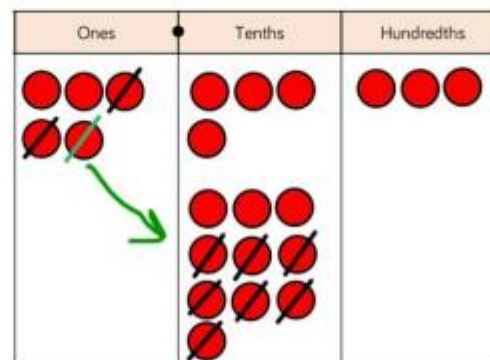
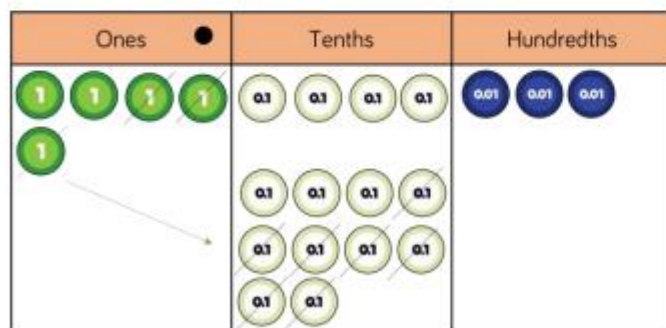
Skill: Subtract with up to 3 decimal places

Year: 5/6



$$\begin{array}{r} 4 \quad 1 \\ 5.43 \\ - 2.7 \\ \hline 2.73 \end{array}$$

$$5.43 - 2.7 = 2.73$$



Place value counters and plain counters on a place value grid are the most effective manipulative when subtracting decimals with 1, 2 and then 3 decimal places.

Ensure children have experience of subtracting decimals with a variety of decimal places. This includes putting this into context when subtracting money and other measures.

Skill: Multiply 4-digit numbers by 2-digit numbers

Year: 5/6

TTh	Th	H	T	O
	2	7	3	9
×			2	8
2	1	9	1	2
₂	₅	₃	₇	
5	4	7	8	0
₁		₁		
7	6	6	9	2
		₁		

$$2,739 \times 28 = 76,692$$

When multiplying 4-digits by 2-digits, children should be confident in using the formal written method.

If they are still struggling with times tables, provide multiplication grids to support when they are focusing on the use of the method.

Consider where exchanged digits are placed and make sure this is consistent.

Skill: Divide multi digits by 2-digits (short division)

Year: 6

		0	3	6
	12	4	4 ₃	7 ₂

$$432 \div 12 = 36$$

$$7,335 \div 15 = 489$$

	0	4	8	9
15	7	7 ₃	13 ₃	13 ₅

15	30	45	60	75	90	105	120	135	150
----	----	----	----	----	----	-----	-----	-----	-----

When children begin to divide up to 4-digits by 2-digits, written methods become the most accurate as concrete and pictorial representations become less effective. Children can write out multiples to support their calculations with larger remainders. Children will also solve problems with remainders where the quotient can be rounded as appropriate.

Skill: Divide multi-digits by 2-digits (long division)

Year: 6

		0	3	6
1	2	4	3	2
	-	3	6	0
			7	2
	-		7	2
				0

(x30)

$$12 \times 1 = 12$$

$$12 \times 2 = 24$$

$$12 \times 3 = 36$$

$$12 \times 4 = 48$$

$$12 \times 5 = 60$$

(x6)

$$12 \times 6 = 72$$

$$12 \times 7 = 84$$

$$12 \times 8 = 96$$

$$12 \times 9 = 108$$

$$12 \times 10 = 120$$

$$432 \div 12 = 36$$

$$7,335 \div 15 = 489$$

	0	4	8	9
15	7	3	3	5
-	6	0	0	0
	1	3	3	5
-	1	2	0	0
		1	3	5
-		1	3	5
				0

(x400)

$$1 \times 15 = 15$$

$$2 \times 15 = 30$$

$$3 \times 15 = 45$$

(x80)

$$4 \times 15 = 60$$

$$5 \times 15 = 75$$

(x9)

$$10 \times 15 = 150$$

Children can also divide by 2-digit numbers using long division.

Children can write out multiples to support their calculations with larger remainders.

Children will also solve problems with remainders where the quotient can be rounded as appropriate.

Skill: Divide multi digits by 2-digits (long division)

Year: 6

$$372 \div 15 = 24 \text{ r}12$$

			2	4	r	1	2
1	5	3	7	2			
	-	3	0	0			
			7	2			
	-		6	0			
			1	2			

- $1 \times 15 = 15$
- $2 \times 15 = 30$
- $3 \times 15 = 45$
- $4 \times 15 = 60$
- $5 \times 15 = 75$
- $10 \times 15 = 150$

When a remainder is left at the end of a calculation, children can either leave it as a remainder or convert it to a fraction. This will depend on the context of the question.

Children can also answer questions where the quotient needs to be rounded according to the context.

			2	4	$\frac{4}{5}$
1	5	3	7	2	
	-	3	0	0	
			7	2	
	-		6	0	
			1	2	

$$372 \div 15 = 24 \frac{4}{5}$$